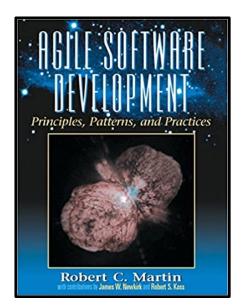
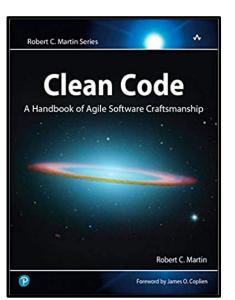
The Sieve of Eratosthenes







Robert Martin 2 @unclebobmartin



2, 3, 5, 7, 11, ...





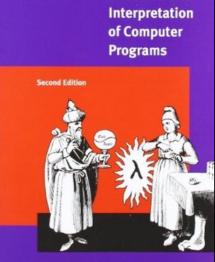




Harold Abelson



Gerald Jay Sussman



Harold Abelson and Gerald Jay Sussman with Julie Sussman

Structure and







In this slide deck we are going to see some examples of how the effort required to read an understand the Sieve of Eratosthenes varies greatly depending on the programming paradigm used to implement the algorithm.

The first version of the sieve that we are going to look at is implemented using imperative and structured programming.

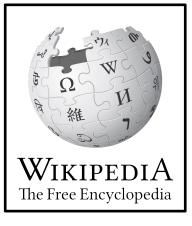
In <u>computer science</u>, **imperative programming** is a <u>programming paradigm</u> of <u>software</u> that uses <u>statements</u> that change a program's <u>state</u>.

Structured programming is a <u>programming paradigm</u> aimed at improving the clarity, quality, and development time of a <u>computer program</u> by making extensive use of the structured control flow constructs of selection (<u>if/then/else</u>) and repetition (<u>while</u> and <u>for</u>), <u>block structures</u>, and <u>subroutines</u>. It is possible to do structured programming in any programming language, though it is preferable to use something like a <u>procedural programming language</u>.

Procedural programming is a <u>programming paradigm</u>, derived from <u>imperative programming</u>,^[1] based on the concept of the <u>procedure call</u>. Procedures (a type of routine or <u>subroutine</u>) simply contain a series of computational steps to be carried out. Any given procedure might be called at any point during a program's execution, including by other procedures or itself. The first major procedural programming languages appeared circa 1957–1964, including Fortran, ALGOL, COBOL, PL/I and BASIC.^[2] Pascal and C were published circa 1970–1972.



The example is on the next slide and is from **Robert Martin's** great book: **Agile Software Development - Principles, Patterns and Practices**.



```
* This class Generates prime numbers up to a user specified
* maximum. The algorithm used is the Sieve of Eratosthenes.
* 
 * Eratosthenes of Cyrene, b. c. 276 BC, Cyrene, Libya --
* d. c. 194, Alexandria. The first man to calculate the
 * circumference of the Earth. Also known for working on
 * calendars with leap years and ran the library at Alexandria.
* 
 * The algorithm is quite simple. Given an array of integers
 * starting at 2. Cross out all multiples of 2. Find the next
 * uncrossed integer, and cross out all of its multiples.
 * Repeat until you have passed the square root of the maximum
 * value.
 * @author Robert C. Martin
 * @version 9 Dec 1999 rcm
 */
public class GeneratePrimes
  /**
   * @param maxValue is the generation limit.
   */
 public static int[] generatePrimes(int maxValue)
While this program makes extensive
use of the control flow constructs of
                                       Principles, Patterns, and Practices
structured programming, it makes
very little use of subroutines.
```

Robert C. Martin

```
public static int[] generatePrimes(int maxValue)
 if (maxValue >= 2) { // the only valid case
   // declarations
   int s = maxValue + 1; // size of array
   boolean[] f = new boolean[s];
    int i;
    // initialize array to true.
   for (i = 0; i < s; i++)</pre>
     f[i] = true;
    // get rid of known non-primes
   f[0] = f[1] = false;
    // sieve
   int j;
   for (i = 2; i < Math.sqrt(s) + 1; i++) {</pre>
     if (f[i]) { // if i is uncrossed, cross its multiples.
        for (j = 2 * i; j < s; j += i)
          f[j] = false; // multiple is not prime
    // how many primes are there?
   int count = 0;
   for (i = 0; i < s; i++) {</pre>
     if (f[i])
        count++; // bump count.
   int[] primes = new int[count];
    // move the primes into the result
   for (i = 0, j = 0; i < s; i++) {</pre>
     if (f[i]) // if prime
        primes[j++] = i;
   return primes; // return the primes
   else // maxValue < 2</pre>
   return new int[0]; // return null array if bad input.
```

This program generates **prime numbers**.

It is **one big function** with many **single letter variables** and **comments** to **help** us read it.

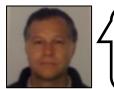


Notice that the generatePrimes function is divided into sections such as declarations, initializations, and sieve.

This is an obvious symptom of doing more than one thing.

Functions that do one thing cannot be reasonably divided into sections.





Instead of showing you the test code associated with that program, here is a simple test demonstrating that the program correctly computes the **primes** in the range 2..30.

```
public class Main {
  public static void main(String[] args) {
    int[] actualPrimes = GeneratePrimes.generatePrimes(30);
    int[] expectedPrimes = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
    if (!Arrays.equals(actualPrimes, expectedPrimes))
        throw new AssertionError(
            "GeneratePrimes.generatePrimes(30) returned " + Arrays.toString(actualPrimes));
    }
}
```





Robert Martin

I wrote the module ... for the first *XP Immersion*.

It was intended to be an example of bad coding and commenting style.

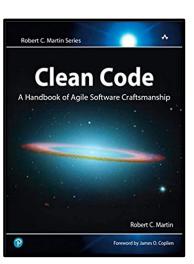
Kent Beck then refactored this code into a much more pleasant form in front of several dozen enthusiastic students.

Later I adapted the example for my book *Agile Software Development, Principles, Patterns, and Practices* and the first of my *Craftsman* articles published in *Software Development* magazine.

What I find fascinating about this module is that <u>there was a time when many of us would have</u> considered it "well documented."

Now we see it as a small mess.

See how many different comment problems you can find.





Jeff Langr



Comments indicate that code is not communicating clearly.

As others have said, "Comments are lies."

You can't trust comments; the only thing you can truly depend on is working code.

Comments are often used as a deodorant for the rotten whiffs of bad code.

Strive to eliminate comments in your code.

becomes superfluous.



Martin Fowler



Robert Martin

The proper use of comments is to compensate for our failure to express ourself in code.

Note that I used the word *failure*. I meant it. Comments are always failures. We must have them because we cannot always figure out how to express ourselves without them, but their use is not a cause for celebration.

When you feel the need to write a comment, first try to refactor the code so that any comment

So when you find yourself in a position where you need to write a **comment**, think it through and see whether there isn't some way to turn the tables and **express yourself in code**.

Every time you express yourself in code, you should pat yourself on the back. Every time you write a comment, you should grimace and feel the failure of your ability of expression.





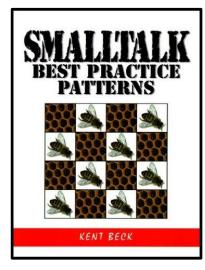


Method Comment Pattern

- How do you comment methods?
- Communicate important information that is not obvious from the code in a comment at the beginning of the method
- I expect you to be skeptical about this pattern
- Experiment:
 - Go through your methods and delete only those comments that duplicate exactly what the code says
 - If you can't delete a comment, see if you can refactor the code using these patterns (...) to communicate the same thing
- I will be willing to bet that when you are done you will have almost no comments left



Of course the above is just a summary of the pattern.







Functions Should Do One Thing

It is often tempting to create <u>functions that have multiple sections that perform a series of operations</u>. <u>Functions of this kind do</u> more than one thing, and should be converted into many smaller functions, each of which does one thing</u>. For example:

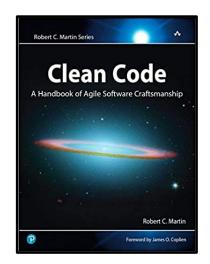
```
public void pay() {
    for (Employee e : employees) {
        if (e.isPayday()) {
            Money pay = e.calculatePay();
            e.deliverPay(pay);
        }
      }
}
```

This bit of code does three things. It loops over all the employees, checks to see whether each employee ought to be paid, and then pays the employee. This code would be better written as:

```
public void pay() {
    for (Employee e : employees)
        payIfNecessary(e);
}
```

```
private void payIfNecessary(Employee e) {
    if (e.isPayday())
        calculateAndDeliverPay(e);
}
```

```
private void calculateAndDeliverPay(Employee e) {
   Money pay = e.calculatePay();
   e.deliverPay(pay);
```









```
Robert Martin
```

Understand the Algorithm

Lots of very funny code is written because people don't take the time to understand the algorithm.

They get something to work by plugging in enough if statements and flags, without really stopping to consider what is really going on.

Programming is often an exploration.

You think you know the right algorithm for something, but then you wind up fiddling with it, prodding and poking at it, until you get it to "work."

How do you know it "works"? Because it passes the test cases you can think of.

There is nothing wrong with this approach.

Indeed, often it is the only way to get a function to do what you think it should.

However, it is not sufficient to leave the quotation marks around the word "work."

Before you consider yourself to be done with a function, make sure you <u>understand</u> how it works.

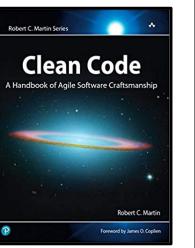
It is not good enough that it passes all the tests. You must <u>know</u>¹⁰ that the solution is <u>correct</u>.

Often the best way to gain this knowledge and understanding is to refactor the function into something that is so clean and expressive that it is obvious how it works.

^{10.} There is a difference between **knowing how the code works** and knowing whether the algorithm will do the job required of it. Being unsure that an algorithm is appropriate is often a fact of life. Being unsure what your code does is just laziness.



Robert Martin





The **prime generator** is hard to **understand** and **maintain** because it consists of a **single method** that **does more than one thing**.

The method is divided into **sections** (each signposted by a **comment**), with each **section** doing one of the **multiple things**.

On the next slide, **Uncle Bob** asserts that if a program is **difficult** to **understand** and/or **change** then it is **broken** and **needs fixing**.

Every software module has three functions.

performs while executing. This function is the reason

First is the function it

for the module's existence.





It takes attention.

It takes discipline.

It takes a passion for creating beauty.

The second function of a module is to afford change.

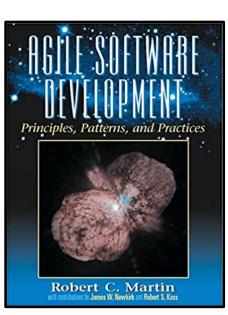
Almost all modules will change in the course of their lives, and it is the responsibility of the developers to make sure that such changes are as simple as possible to make.

A module that is difficult to change is broken and needs fixing, even though it works.

The third function of a module is to communicate to its readers.

Developers who are not familiar with the module should be able to read and understand it without undue mental gymnastics.

A module that does not communicate is broken and needs to be fixed.



What does it take to make a module easy to read and easy to change?

Much of this book is dedicated to principles and patterns whose primary goal is to help you create modules that are flexible and adaptable.

But it takes something more than just principles and patterns to make a module that is easy to read and change.

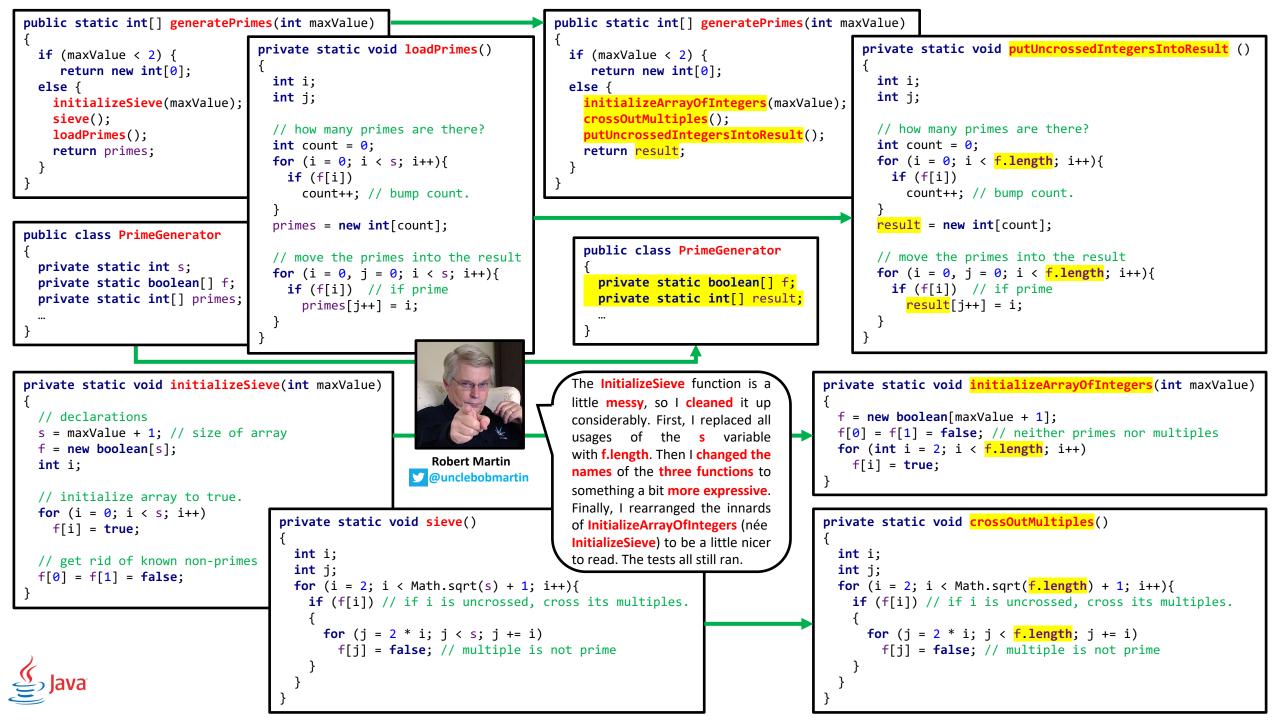


What we are going to do next is follow Uncle Bob as he refactors the prime generator so that it becomes easier to understand and to change.

```
public static int[] generatePrimes(int maxValue)
         * This class Generates prime numbers up to a user specified
         * maximum. The algorithm used is the Sieve of Eratosthenes.
                                                                                   if (maxValue >= 2) { // the only valid case
                                                                                     // declarations
         * 
                                                                                     int s = maxValue + 1; // size of array
         * Eratosthenes of Cyrene, b. c. 276 BC, Cyrene, Libya --
         * d. c. 194, Alexandria. The first man to calculate the
                                                                                     boolean[] f = new boolean[s];
         * circumference of the Earth. Also known for working on
                                                                                     int i;
         * calendars with leap years and ran the library at Alexandria.
                                                                                     // initialize array to true.
         * 
         * The algorithm is quite simple. Given an array of integers
                                                                                     for (i = 0; i < s; i++)</pre>
         * starting at 2. Cross out all multiples of 2. Find the next
                                                                                       f[i] = true;
         * uncrossed integer, and cross out all of its multiples.
         * Repeat until you have passed the square root of the maximum
                                                                                     // get rid of known non-primes
                                                                                     f[0] = f[1] = false;
         * value.
         * @author Robert C. Martin
                                                                                     // sieve
         * @version 9 Dec 1999 rcm
                                                                                     int j;
                                                                                     for (i = 2; i < Math.sqrt(s) + 1; i++) {</pre>
                                                                                       if (f[i]) { // if i is uncrossed, cross its multiples.
        public class GeneratePrimes
                                                                                         for (j = 2 * i; j < s; j += i)</pre>
                                                                                           f[j] = false; // multiple is not prime
            * @param maxValue is the generation limit.
                                                                                     }
          public static int[] generatePrimes(int maxValue)
                                                                                     // how many primes are there?
                                                                                     int count = 0;
                                                                                     for (i = 0; i < s; i++) {</pre>
                                                                                       if (f[i])
                                                                                          count++; // bump count.
                                                                                     int[] primes = new int[count];
                      It seems pretty clear that the main function wants to be
                                                                                     // move the primes into the result
                      three separate functions.
                                                                                     for (i = 0, j = 0; i < s; i++) {</pre>
                                                                                       if (f[i]) // if prime
                      The first initializes all the variables and sets up the sieve.
                                                                                         primes[j++] = i;
                                                                                     }
                      The second executes the sieve, and the third loads the
                                                                                     return primes; // return the primes
                      sieved results into an integer array.
                                                                                   } else // maxValue < 2</pre>
Robert Martin
                                                                                     return new int[0]; // return null array if bad input.
@unclebobmartin
```

```
لے ا
```

```
public static int[] generatePrimes(int maxValue)
                                                                     public class PrimeGenerator
                                                                                                                              public static int[] generatePrimes(int maxValue)
 if (maxValue >= 2) { // the only valid case
                                                                      private static int s;
                                                                                                                                if (maxValue < 2) {</pre>
   // declarations
                                                                       private static boolean[] f;
                                                                                                                                  return new int[0];
   int s = maxValue + 1; // size of array
                                                                       private static int[] primes;
                                                                                                                                else {
   boolean[] f = new boolean[s];
                                                                                                                                  initializeSieve(maxValue);
    int i;
                                                                       public static int[] generatePrimes(int maxValue)
                                                                                                                                  sieve();
                                                                                                                                  loadPrimes();
   // initialize array to true.
                                                                                                                                  return primes;
   for (i = 0; i < s; i++)</pre>
     f[i] = true;
   // get rid of known non-primes
                                                                                                                               To expose this structure more clearly, I extracted those
                                                                    private static void initializeSieve(int maxValue)
   f[0] = f[1] = false;
                                                                                                                               functions into three separate methods.
                                                                       // declarations
    // sieve
                                                                       s = maxValue + 1; // size of array
                                                                                                                               I also removed a few unnecessary comments and
   int j;
                                                                       f = new boolean[s];
                                                                                                                               changed the name of the class to PrimeGenerator.
   for (i = 2; i < Math.sqrt(s) + 1; i++) {</pre>
                                                                       int i;
     if (f[i]) { // if i is uncrossed, cross its multiples.
        for (j = 2 * i; j < s; j += i)
                                                                                                                               The tests all still ran.
                                                                       // initialize array to true.
          f[j] = false; // multiple is not prime
                                                                      for (i = 0; i < s; i++)</pre>
                                                                                                                                          private static void loadPrimes()
                                                                        f[i] = true;
                                                                                                                                            int i;
                                                                       // get rid of known non-primes
    // how many primes are there?
                                                                                                            Robert Martin
                                                                                                                                            int j;
                                                                       f[0] = f[1] = false;
   int count = 0;
                                                                                                        🥑 @unclebobmartin
   for (i = 0; i < s; i++) {</pre>
                                                                                                                                            // how many primes are there?
     if (f[i])
                                                                                                                                             int count = 0;
        count++; // bump count.
                                                                                                    Extracting the three functions forced
                                                                                                                                            for (i = 0; i < s; i++){</pre>
                                                                                                    me to promote some of the variables
                                                                                                                                              if (f[i])
   int[] primes = new int[count];
                                                                                                    of the function to static fields of the
                                                                                                                                                 count++; // bump count.
                                                                    private static void sieve()
                                                                                                    class. This makes it much clearer which
   // move the primes into the result
                                                                                                                                            primes = new int[count];
                                                                                                    variables are local and which have
   for (i = 0, j = 0; i < s; i++) {
                                                                       int i;
                                                                                                    wider influence.
     if (f[i]) // if prime
                                                                                                                                            // move the primes into the result
                                                                       int j;
        primes[j++] = i;
                                                                                                                                            for (i = 0, j = 0; i < s; i++)
                                                                      for (i = 2; i < Math.sqrt(s) + 1; i++){</pre>
                                                                        if (f[i]) // if i is uncrossed, cross its multiples.
                                                                                                                                              if (f[i]) // if prime
                                                                                                                                                 primes[j++] = i;
   return primes; // return the primes
                                                                          for (j = 2 * i; j < s; j += i)
                                                                             f[i] = false: // multiple is not prime
 } else // maxValue < 2</pre>
   return new int[0]; // return null array if bad input.
```



Next, I looked at **crossOutMultiples**. There were a number of statements in this function, and in others, of the form **if(f[i] == true)**. The intent was to check whether **i** was **uncrossed**, so I changed the name of **f** to **unCrossed**. But this led to ugly statements, such as **unCrossed[i] = false**. I found the **double negative** confusing. So I changed the name of the array to **isCrossed** and changed the sense of all the Booleans. The tests all still ran.



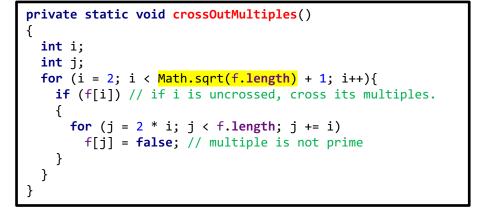
I got rid of the initialization that set isCrossed[0] and isCrossed[1] to true and simply made sure that no part of the function used the isCrossed array for indexes less than 2. I extracted the inner loop of the crossOutMultiples function and called it crossOutMultiplesOf. I also thought that if (isCrossed[i] == false) was confusing, so I created a function called notCrossed and changed the if statement to if (notCrossed(i)). The tests all still ran.

Robert Martin

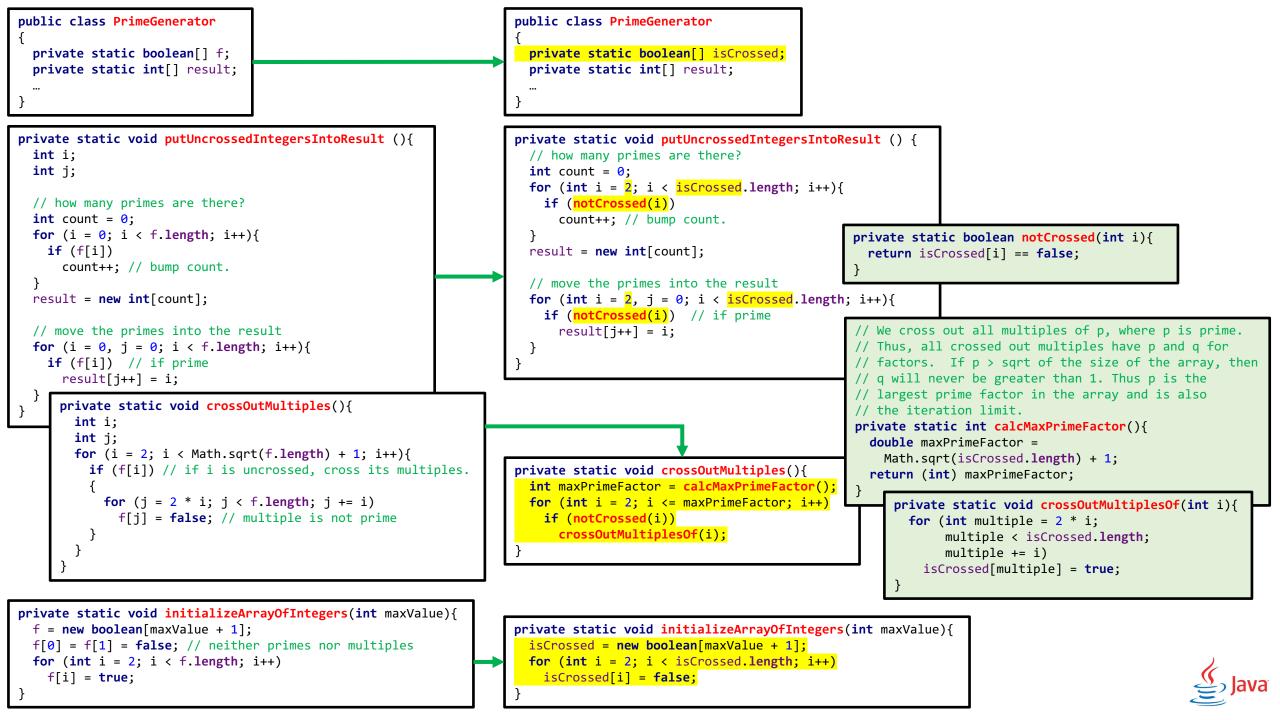
I spent a bit of time writing a comment that tried to explain why you have to iterate only up to the square root of the array size. This led me to extract the calculation into a function where I could put the explanatory comment. In writing the comment, I realized that the square root is the maximum prime factor of any of the integers in the array. So I chose that name for the variables and functions that dealt with it. The result of all these refactorings are [on the next page] ... The tests all still ran.

```
private static void crossOutMultiples()
{
    int i;
    int j;
    for (i = 2; i < Math.sqrt(f.length) + 1; i++){
        if (f[i]) // if i is uncrossed, cross its multiples.
        {
            for (j = 2 * i; j < f.length; j += i)
                 f[j] = false; // multiple is not prime
        }
    }
}</pre>
```

```
private static void initializeArrayOfIntegers(int maxValue)
{
    f = new boolean[maxValue + 1];
    f[0] = f[1] = false; // neither primes nor multiples
    for (int i = 2; i < f.length; i++)
        f[i] = true;
}</pre>
```







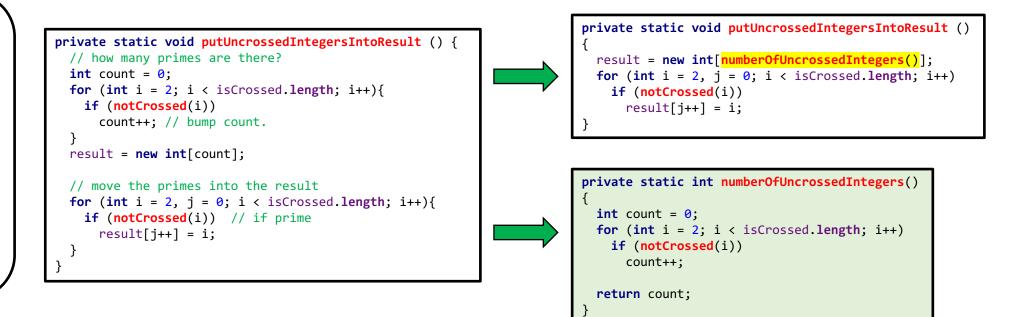
The last function to refactor is **PutUncrossedIntegersIntoResult**. This method has **two parts**.

The **first** counts the number of uncrossed integers in the array and creates the result array of that size.

The **second** moves the uncrossed integers into the result array.

I extracted the first part into its own function and did some miscellaneous cleanup.

The tests all still ran.











Robert Martin @unclebobmartin Next, I made one final pass over the whole program, reading it from beginning to end, rather like one would read a geometric proof.

This is an important step.

So far, I've been refactoring fragments.

Now I want to see whether the whole program hangs together as a *readable* whole.



Robert Martin

First, I realize that I don't like the name InitializeArrayOfIntegers.

What's being initialized is not, in fact, an array of integers but an array of Booleans.

But InitializeArrayOfBooleans is not an improvement.

What we are really doing in this method is **uncrossing** all the relevant integers so that we can then **cross out** the **multiples**.

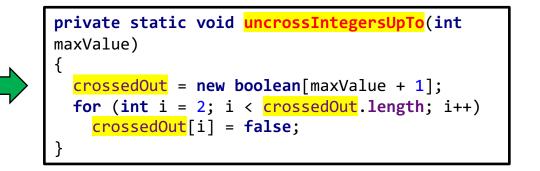
So I change the name to **uncrossIntegersUpTo**.

I also realize that I don't like the name **isCrossed** for the array of Booleans.

So I change it to **crossedOut**.

The tests all still run.

```
private static void
initializeArrayOfIntegers(int maxValue)
{
    isCrossed = new boolean[maxValue + 1];
    for (int i = 2; i < isCrossed.length; i++)
        isCrossed[i] = false;
```







```
Robert Martin
@unclebobmartin
```

One might think that I'm being **frivolous** with these **name changes**, but with a refactoring browser, you can afford to do these kinds of **tweaks**; they cost virtually nothing.

Even without a refactoring browser, a simple search and replace is pretty cheap.

And the tests strongly mitigate any chance that we might unknowingly break something.





I don't know what I was smoking when I wrote all that maxPrimeFactor stuff.

Yikes! The square root of the size of the array is not necessarily prime.

That method did *not* calculate the **maximum prime factor**.

The explanatory comment was simply wrong.

So I rewrote the comment to better explain the rationale behind the square root and rename all the variables appropriately.

The tests all still run.

```
private static void crossOutMultiples()
{
    int maxPrimeFactor = calcMaxPrimeFactor();
    for (int i = 2; i <= maxPrimeFactor; i++)
        if (notCrossed(i))
            crossOutMultiplesOf(i);
}</pre>
```

private static void crossOutMultiples() int limit = determineIterationLimit(); for (int i = 2; i <= limit; i++)</pre> if (notCrossed(i)) crossOutMultiplesOf(i);



// We cross out all multiples of p, where p is prime. // Thus, all crossed out multiples have p and q for // factors. If p > sqrt of the size of the array, then // q will never be greater than 1. Thus p is the // largest prime factor in the array and is also // the iteration limit. private static int calcMaxPrimeFactor() { double maxPrimeFactor = Math.sqrt(crossedOut.length) + 1; return (int) maxPrimeFactor;

// Every multiple in the array has a prime factor that
// is less than or equal to the root of the array size,
// so we don't have to cross off multiples of numbers
// larger than that root.
private static int determineIterationLimit()

double iterationLimit = Math.sqrt(crossedOut.length) + 1; return (int) iterationLimit; What the devil is that +1 doing in there?



Robert Martin

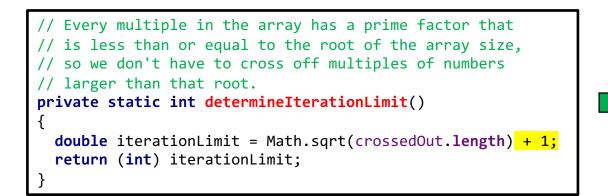
It must have been paranoia.

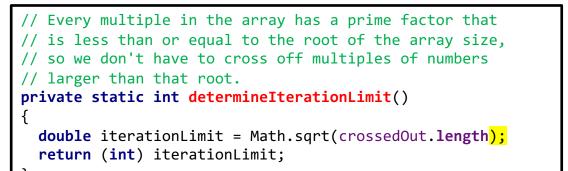
I was afraid that a **fractional square root** would convert to an integer that was too small to serve as the iteration limit.

But that's silly.

The true iteration limit is the largest **prime** less than or equal to the **square root** of the size of the array.

I'll get rid of the +1.







The tests all run, but that last change makes me pretty nervous.

I understand the rationale behind the square root, but I've got a nagging feeling that there may be some corner cases that aren't being covered.

So I'll write another test that checks that there are no **multiples** in any of the **prime** lists between 2 and 500. ...

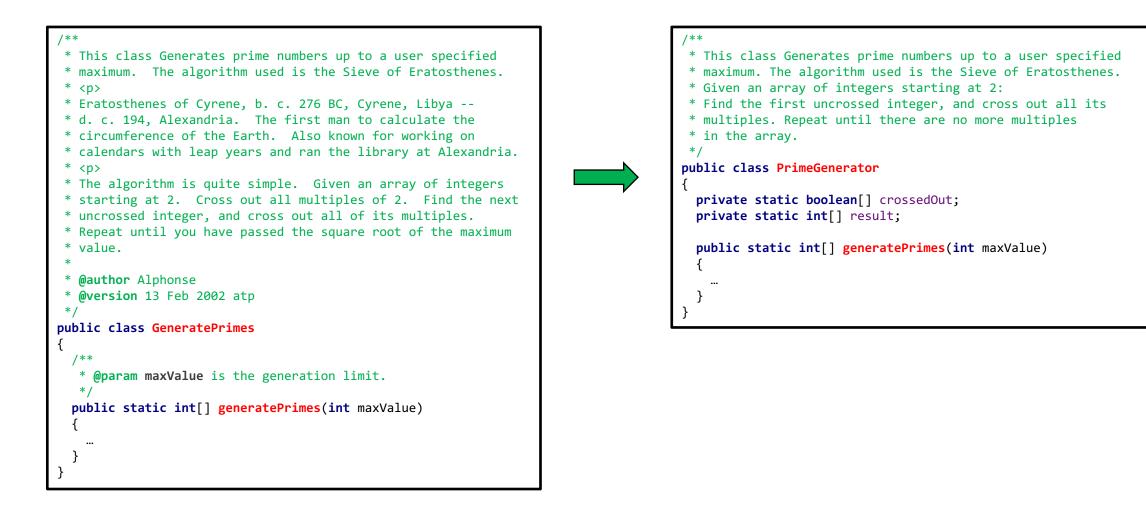
The new test passes, and my fears are allayed.



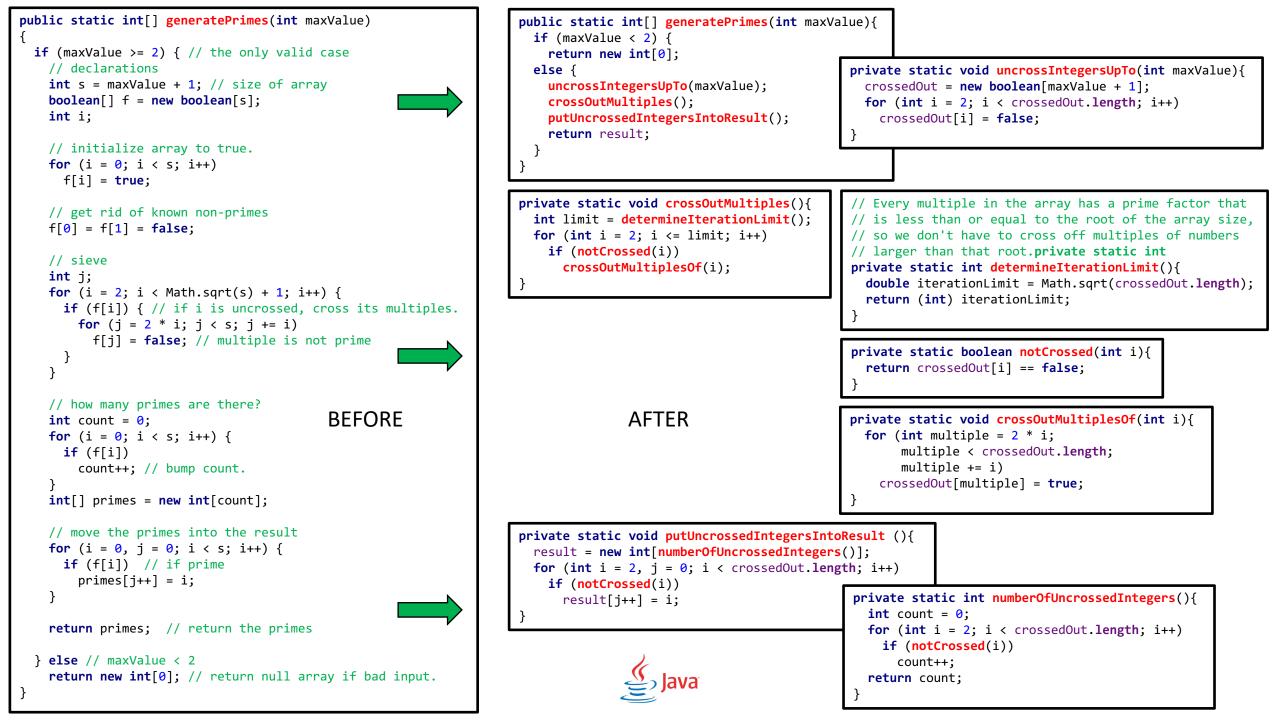
private static void uncrossIntegersUpTo(int maxValue) * This class Generates prime numbers up to a user specified * maximum. The algorithm used is the Sieve of Eratosthenes. crossedOut = new boolean[maxValue + 1]; * Given an array of integers starting at 2: for (int i = 2; i < crossedOut.length; i++)</pre> * Find the first uncrossed integer, and cross out all its crossedOut[i] = false; * multiples. Repeat until there are no more multiples * in the array. private static void crossOutMultiples() // Every multiple in the array has a prime factor that public class PrimeGenerator // is less than or equal to the root of the array size, // so we don't have to cross off multiples of numbers int limit = determineIterationLimit(); private static boolean[] crossedOut; // larger than that root. for (int i = 2; i <= limit; i++)</pre> private static int[] result; private static int determineIterationLimit() if (notCrossed(i)) crossOutMultiplesOf(i); public static int[] generatePrimes(int maxValue) double iterationLimit = Math.sqrt(crossedOut.length); return (int) iterationLimit; if (maxValue < 2) {</pre> return new int[0]; The end result of this program reads else { private static boolean notCrossed(int i) much better than it did at the start. It uncrossIntegersUpTo(maxValue); crossOutMultiples(); also works a bit better. I'm pretty return crossedOut[i] == false; putUncrossedIntegersIntoResult(); pleased with the outcome. The return result; program is much easier to understand and is therefore much easier to private static void crossOutMultiplesOf(int i) change. Also, the structure of the for (int multiple = 2 * i; program has isolated its parts from multiple < crossedOut.length;</pre> one another. This also makes the multiple += i) crossedOut[multiple] = true; program much easier to change. Note that the use of comments is significantly restrained. private static void putUncrossedIntegersIntoResult () There are just two comments in private static int numberOfUncrossedIntegers() result = new int[numberOfUncrossedIntegers()]; the whole module. for (int i = 2, j = 0; i < crossedOut.length; i++)</pre> if (notCrossed(i)) **int** count = 0; for (int i = 2; i < crossedOut.length; i++)</pre> **Both** comments are explanatory result[j++] = i; if (notCrossed(i)) in nature. count++; **Robert Martin** return count; @unclebobmartin

BEFORE

AFTER







The original **sieve** program was developed using **imperative programming** and, except for the fact that it consisted of a single method, **structured programming**. It was then **refactored**, using **functional decomposition**, into a **more understandable** and **maintainable** program which, consisting of several methods, could now more legitimately be considered an example of **structured/procedural programming**.



What we are going to do next is look at a **sieve** program developed using the following:

- 1. The immutable FP data structure of a sequence, implemented using a list
- 2. The basic sequence operations to
 - Construct a sequence
 - Get the **first** element of a **sequence**
 - Get the **rest** of a **sequence**
- 3. A filter function that given a sequence and a predicate (a function that given a value, returns true if the value satisfies the predicate and false otherwise), returns a new sequence by selecting only those elements of the original sequence that satisfy the predicate.

What we'll find is that using these simple building blocks it is possible to write a sieve program which is so simple that it is much easier to understand and maintain than the procedural one we have just seen.

By the way, don't assume that **Uncle Bob** isn't interested in alternative ways of implementing the **Sieve of Eratosthenes**, as we shall see in part 2.

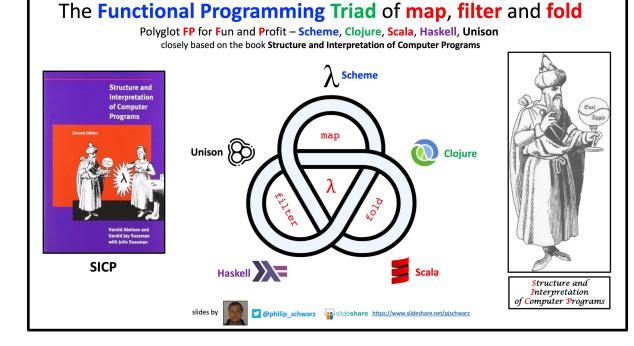




This simpler **sieve** program is described in **Structure and Interpretation of Computer Programs (SICP)** using **Scheme**.

The following two slides are a lightning-fast, extremely minimal refresher on the **building blocks** used to develop the program.

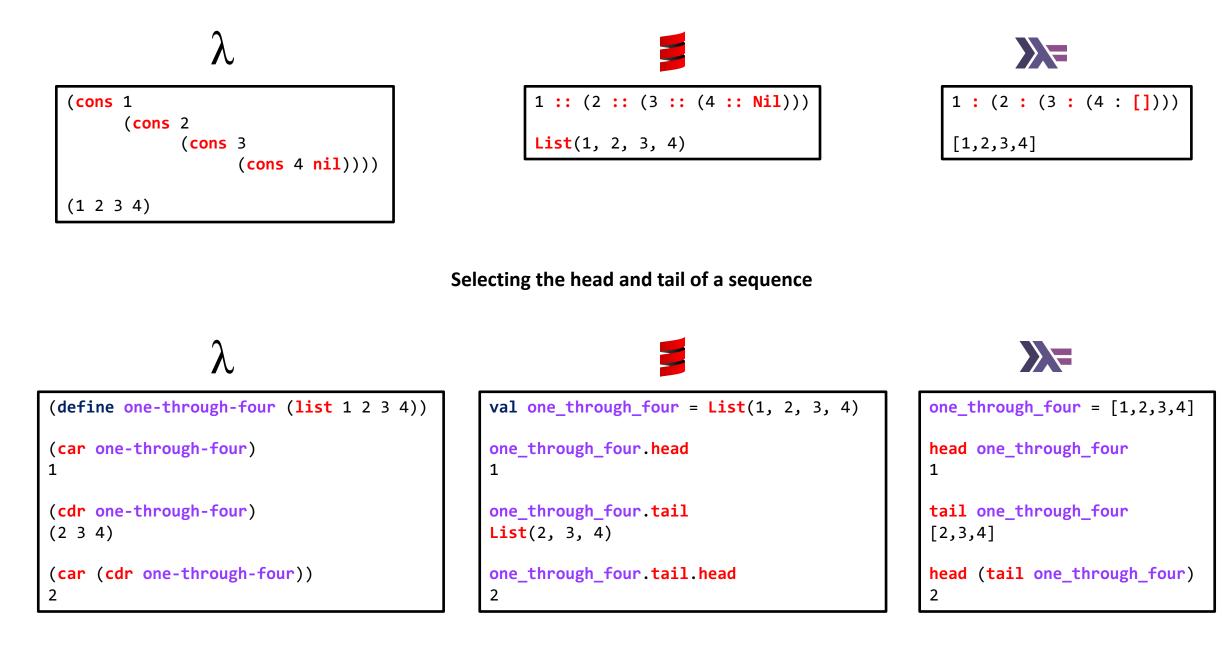
If you are completely new to **immutable data structures**, **sequences**, and **filtering**, and could do with an introduction to them, then why not catch up using the slide deck below?



Structure and Interpretation of Computer Programs Second Edition Harold Abelson and **Gerald Jay Sussman** with Julie Sussman

SICP

Constructing a sequence



λ	<pre>(define (filter predicate sequence) (cond ((null? sequence) nil) ((predicate (car sequence)) (cons (car sequence) (filter predicate (cdr sequence)))) (else (filter predicate (cdr sequence)))))</pre>	<pre>scheme> (filter odd? (list 1 2 3 4 5)) (1 3 5)</pre>
	<pre>def filter[A](predicate: A => Boolean, sequence: List[A]): List[A] = sequence match case Nil => Nil case x::xs => if predicate(x) then x::filter(predicate,xs) else filter(predicate,xs)</pre>	<pre>def isOdd(n: Int): Boolean = n % 2 == 1 scala> List(1, 2, 3, 4, 5).filter(isOdd) val res0: List[Int] = List(1, 3, 5)</pre>
>>	<pre>filter :: (a -> Bool) -> [a] -> [a] filter _ [] = [] filter predicate (x:xs) = if (predicate x)</pre>	<pre>is_odd :: Int -> Boolean is_odd n = (mod n 2) == 1 haskell> filter is_odd [1,2,3,4,5] [1,3,5]</pre>



What I said earlier wasn't the full truth: the **sieve** program in **SICP** uses not just plain **sequences**, implemented using **lists**, but ones implemented using **streams**, i.e. **lazy** and possibly **infinite** sequences.

An introduction to **streams** is outside the scope of this deck so let's learn (or review) just enough about **streams** to be able to understand the **SICP sieve** program, so that we can then convert the program to use **lists** rather than **streams**. Streams are a clever idea that allows one to use sequence manipulations without incurring the costs of manipulating sequences as lists.

With streams we can achieve the best of both worlds: We can formulate programs elegantly as sequence manipulations, while attaining the efficiency of incremental computation.

The basic idea is to arrange to construct a stream only partially, and to pass the partial construction to the program that consumes the stream.

If the consumer attempts to access a part of the stream that has not yet been constructed, the stream will automatically construct just enough more of itself to produce the required part, thus preserving the illusion that the entire stream exists.

In other words, although we will write programs as if we were processing complete sequences, we design our stream implementation to automatically and transparently interleave the construction of the stream with its use.

On the surface, streams are just lists with different names for the procedures that manipulate them.

There is a constructor, cons-stream, and two selectors, stream-car and stream-cdr, which satisfy the constraints

```
(stream-car (cons-stream x y)) = x
```

```
(stream-cdr (cons-stream x y)) = y
```

There is a distinguishable object, the-empty-stream, which cannot be the result of any cons-stream operation, and which can be identified with the predicate stream-null?. Thus we can make and use streams, in just the same way as we can make and use lists, to represent aggregate data arranged in a sequence.



Structure and Interpretation of Computer Programs



To compute the **prime numbers**, we take the **infinite stream** of integers from 2 onwards and pass them through a **sieve**.

(define primes (sieve (integers-starting-from 2)))

Sieving a **stream** of integers so that we only keep those that are **prime** numbers is done by constructing a **new stream** as follows:

- The head of the sieved stream is the head of the incoming stream. Let's refer to this as the next prime number. See next slide for why this is a prime number.
- 2. The tail of the sieved stream is created by recursively sieving a new stream which is the result of taking the tail of the incoming stream and then filtering out any integers which are multiples of the next prime number and which are therefore not prime.



The way the **sieve** function works out which integers are **multiples** of the **next prime number** (so that it can filter them out), is by using the **divisible**? function to check that they are not **divisible** by the **prime number**.

define (<mark>sieve</mark> stream)			
(cons-stream			
(stream-car stream)			
(sieve (stream-filter			
<pre>(lambda (x)(not (divisible?</pre>	х	(stream-car	<pre>stream))</pre>
<pre>(stream-cdr stream)))))</pre>			

(define (divisible? x y)
 (= (remainder x y) 0))

<pre>scheme></pre>	<pre>(divisible?</pre>	63)	
#t			

scheme> (divisible? 6 4)
#f



As for the infinite integers from 2 onwards, they are defined **recursively**.

(define (integers-starting-from n)
 (cons-stream n (integers-starting-from (+ n 1))))

We start with the integers beginning with 2, which is the first prime.

To get the rest of the **primes**, we start by **filtering** the **multiples** of 2 from the **rest** of the integers.

This leaves a **stream** beginning with 3, which is the **next prime**.

Now we filter the **multiples** of 3 from the rest of this **stream**.

This leaves a **stream** beginning with 5, which is the **next prime**, and so on. In other words, we construct the **primes** by a **sieving process**, described as follows: To **sieve** a **stream** S, form a **stream** whose first element is the first element of S and the rest of which is obtained by **filtering** all **multiples** of the first element of S out of the rest of S and **sieving** the result.

This process is readily described in terms of stream operations:

```
(define primes (sieve (integers-starting-from 2)))
```

Now to find a particular **prime** we need only ask for it:

```
scheme> (stream-ref primes 50)
233
```

```
(define (stream-ref s n)
 (if (= n 0)
    (stream-car s)
    (stream-ref (stream-cdr s) (- n 1))))
```



Structure and Interpretation of Computer Programs

λ

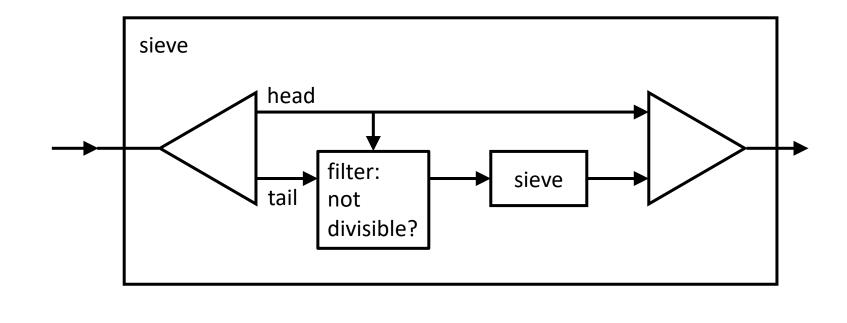
It is interesting to contemplate the signal-processing system set up by sieve, shown in the "Henderson diagram" in figure 3.32.

The **input stream** feeds into an "**unconser**" that separates the **first** element of the **stream** from the **rest** of the **stream**.

The first element is used to construct a **divisibility filter**, through which the **rest** is passed, and the **output** of the filter is fed to **another sieve box**.

Then the original **first element** is **consed** onto the **output** of the **internal sieve** to form the **output stream**.

Thus, not only is the stream infinite, but the signal processor is also infinite, because the sieve contains a sieve within it.





Programs must be written for people to read, and only incidentally for machines to execute.



Harold Abelson

The key to understanding complicated things is knowing what not to look at.



Gerald Jay Sussman



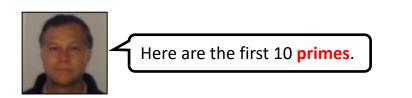




Because the stream of primes is infinite, some auxiliary functions are needed to display a subset of them.

```
(define (stream-take stream n)
  (if (= n 0)
    nil
    (cons-stream
        (stream-car stream)
        (stream-take (stream-cdr stream) (- n 1)))))
(define (display-stream s)
  (stream-for-each display-line s))
(define (display-line x)
  (newline)
  (display x))
```

Λ



scheme>	(display-stream	(stream-take	primes	10))
2				
3				
5				
7				
11				
13				
17				
19				
23				
29				



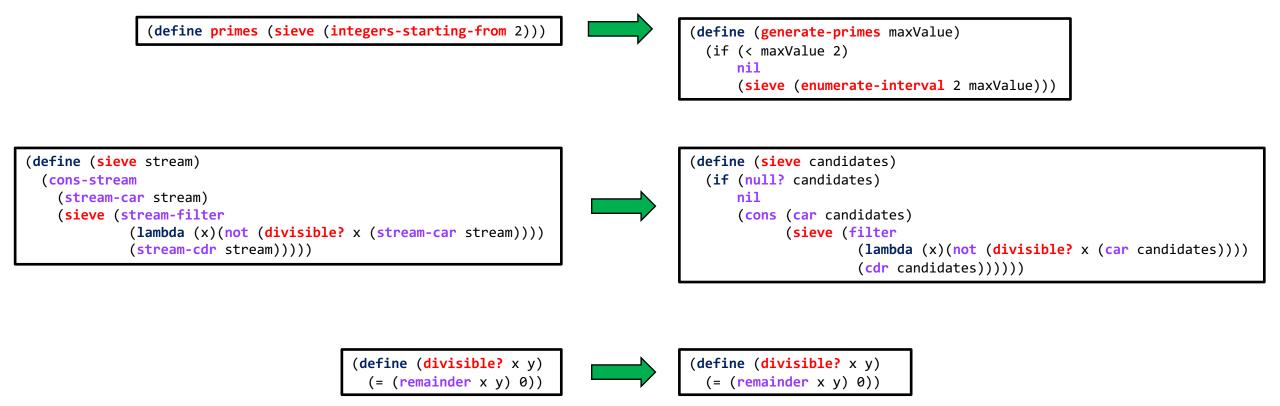
```
/**
```

```
* This class Generates prime numbers up to a user specified
* maximum. The algorithm used is the Sieve of Eratosthenes.
* Given an array of integers starting at 2:
* Find the first uncrossed integer, and cross out all its
* multiples. Repeat until there are no more multiples
* in the array.
*/
public class PrimeGenerator
{
    private static boolean[] crossedOut;
    private static int[] result;
    public static int[] generatePrimes(int maxValue)
    {
        ...
    }
}
```



On the next slide we take the **Scheme sieve** program and modify it as follows:

- get the program to operate on **finite lists** rather than **infinite streams**.
- replace the **primes** function with with a **generatePrimes** function.



(define (integers-starting-from n)
 (cons-stream n (integers-starting-from (+ n 1))))



(define (enumerate-interval low high)
 (if (> low high)
 nil
 (cons low (enumerate-interval (+ low 1) high))))

(define (generate-primes maxValue) (if (< maxValue 2) nil (sieve (enumerate-interval 2 maxValue)))
Here is the resulting program again. Let's take it for a spin.
(define (sieve candidates) (if (null? candidates) nil (cons (car candidates) (sieve (filter)
(define (enumerate-interval low high) nil (cons low (enumerate-interval (+ low 1) high))))

(lambda (x)(not (divisible? x (car candidates))))
(cdr candidates)))))

(define (divisible? x y)
 (= (remainder x y) 0))

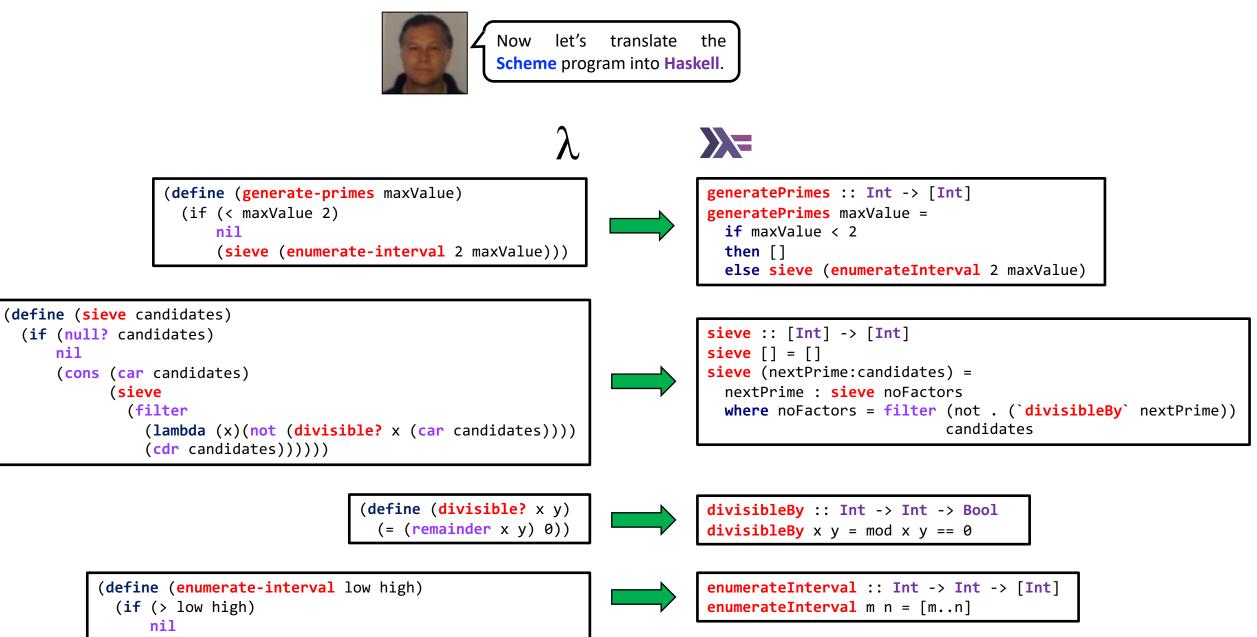
scheme> (sieve '(2 3 4 5 6 7 8 9 10))
(2 3 5 7)

scheme> (enumerate-interval 2 10)
(2 3 4 5 6 7 8 9 10)

scheme> (sieve (enumerate-interval 2 10))
(2 3 5 7)

scheme> (generate-primes 10)
(2 3 5 7)

scheme> (generate-primes 30)
(2 3 5 7 11 13 17 19 23 29)



(cons low (enumerate-interval (+ low 1) high))))



Here is the **Haskell**, program again, after inlining the **enumerateInterval** function. Let's take it for a spin.

```
generatePrimes :: Int -> [Int]
generatePrimes maxValue =
    if maxValue < 2
    then []
    else sieve [2..maxValue]</pre>
```

divisibleBy :: Int -> Int -> Bool divisibleBy x y = mod x y == 0

haskell> <mark>generatePrimes</mark> 30	
[2,3,5,7,11,13,17,19,23,29]	



Haskell is lazy e.g. its lists can be infinite, so just like we did in Scheme with streams, we could define the infinite list of primes as the sieve of the infinite list of integers starting from 2, and then operate on the primes.



(define primes (sieve (integers-starting-from 2)))

primes = sieve [2..]

scheme>	(display-stream	(stream-take	primes	10))
2				
3				
5				
7				
11				
13				
17				
19				
23				
29				

haskell> take 10 primes [2,3,5,7,11,13,17,19,23,29]

scheme> (stream-ref primes 50)
233

haskell>	primes	!!	50
233			





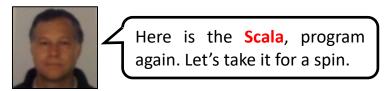
def generatePrimes(maxValue: Int): List[Int] =
 if maxValue < 2
 then Nil
 else sieve(List.range(2,maxValue + 1))</pre>

<pre>def sieve(candidates: List[Int]): List[Int] = candidates match</pre>
case Nil => Nil
<pre>case nextPrime :: rest =></pre>
val nonMultiples = rest <mark>filterNot</mark> (_ divisibleBy nextPrime) nextPrime :: <mark>sieve</mark> (nonMultiples)

divisibleBy :: Int -> Int -> Bool divisibleBy x y = mod x y == 0



extension (m: Int)
 def divisibleBy(n: Int): Boolean = m % n == 0



```
def generatePrimes(maxValue: Int): List[Int] =
    if maxValue < 2
    then Nil
    else sieve(List.range(2,maxValue + 1))</pre>
```

```
def sieve(candidates: List[Int]): List[Int] = candidates match
    case Nil => Nil
    case nextPrime :: rest =>
    val nonMultiples = rest filterNot (_ divisibleBy nextPrime)
    nextPrime :: sieve(nonMultiples)
```

extension (m: Int)
 def divisibleBy(n: Int): Boolean = m % n == 0

```
scala> generatePrimes(30)
val res0: List[Int] = List(2, 3, 5, 7, 11, 13, 17, 19, 23, 29)
```



In the next two slides, let's compare the **Scala** program with the **Java** program, both before and after refactoring the latter.

```
public static int[] generatePrimes(int maxValue)
 if (maxValue >= 2) { // the only valid case
   // declarations
   int s = maxValue + 1; // size of array
   boolean[] f = new boolean[s];
   int i;
   // initialize array to true.
   for (i = 0; i < s; i++)</pre>
     f[i] = true;
   // get rid of known non-primes
   f[0] = f[1] = false;
    // sieve
    int j;
   for (i = 2; i < Math.sqrt(s) + 1; i++) {</pre>
     if (f[i]) { // if i is uncrossed, cross its multiples.
       for (j = 2 * i; j < s; j += i)
          f[j] = false; // multiple is not prime
     }
    }
    // how many primes are there?
    int count = 0;
   for (i = 0; i < s; i++) {</pre>
     if (f[i])
        count++; // bump count.
   int[] primes = new int[count];
   // move the primes into the result
   for (i = 0, j = 0; i < s; i++) {
     if (f[i]) // if prime
        primes[j++] = i;
    }
    return primes; // return the primes
 } else // maxValue < 2</pre>
    return new int[0]; // return null array if bad input.
```

```
FP immutable
sequence and
filtering
```



If I have to choose which of the two programs I'd rather have to **understand** and **maintain**, then I am compelled to pick the one on the right, due to its **succinctness**.



def generatePrimes(maxValue: Int): List[Int] =
 if maxValue < 2
 then Nil
 else sieve(List.range(2, maxValue + 1))</pre>

def sieve(candidates: List[Int]): List[Int] = candidates match
 case Nil => Nil
 case nextPrime :: rest =>
 val nonMultiples = rest filterNot (_ divisibleBy nextPrime)
 nextPrime :: sieve(nonMultiples)

extension (m: Int)
 def divisibleBy(n: Int): Boolean =
 m % n == 0

imperative and structured programming

<pre>public static int[] generatePrimes(int max\ if (maxValue < 2) { return new int[0]; else { uncrossIntegersUpTo(maxValue); } }</pre>	<pre>/alue){ for the static void uncrossIntegersUpTo(int maxValue){</pre>	FP immutable sequence and filtering
<pre>crossOutMultiples(); putUncrossedIntegersIntoResult(); return result; } </pre>	<pre>crossedOut = new boolean[maxValue + 1]; for (int i = 2; i < crossedOut.length; i++) crossedOut[i] = false; }</pre>	
<pre>private static void crossOutMultiples(){ int limit = determineIterationLimit(); for (int i = 2; i <= limit; i++) if (notCrossed(i)) crossOutMultiplesOf(i); } </pre>	<pre>// Every multiple in the array has a prime factor that // is less than or equal to the root of the array size, // so we don't have to cross off multiples of numbers // larger than that root.private static int private static int determineIterationLimit(){ double iterationLimit = Math.sqrt(crossedOut.length);</pre>	<pre>def generatePrimes(maxValue: Int): List[Int] = if maxValue < 2 then Nil else sieve(List.range(2,maxValue + 1))</pre>
	<pre>return (int) iterationLimit; } private static boolean notCrossed(int i){ return crossedOut[i] == false; }</pre>	<pre>def sieve(candidates: List[Int]): List[Int] = candidates match case Nil => Nil case nextPrime :: rest => val nonMultiples = rest filterNot (_ divisibleBy nextPrime) nextPrime :: sieve(nonMultiples)</pre>
procedural programming	<pre>private static void crossOutMultiplesOf(int i){ for (int multiple = 2 * i; multiple < crossedOut.length; multiple += i) crossedOut[multiple] = true; }</pre>	<pre>extension (m: Int) def divisibleBy(n: Int): Boolean = m % n == 0</pre>
<pre>private static void putUncrossedIntegersInt result = new int[numberOfUncrossedInteger for (int i = 2, j = 0; i < crossedOut.ler if (notCrossed(i))</pre>	rs()];	Althought the program on the left is easier to understand and maintain thatn the original in the previous slide, the
<pre>result[j++] = i; }</pre>	<pre>private static int numberOfUncrossedIntegers(){ int count = 0; for (int i = 2; i < crossedOut.length; i++) if (notCrossed(i)) count++; return count; } </pre>	succinctness of the program on the right still makes the latter my preferred choice for understanding and maintenance.

