The Sieve of Eratosthe

Part 1

2, 3, 5, 7, 11, …

Robert Martin @unclebobmartin

In this slide deck we are going to see some examples of how the **effort required** to greatly depending on the **programming [paradigm](https://en.wikipedia.org/wiki/Programming_paradigm)** used to implement the **[algorithm](https://en.wikipedia.org/wiki/Imperative_programming)**.

The first version of the **sieve** that we are going to look at is implemented using *imperat*

In computer science, **imperative programming** is a programming paradigm of software change a program's state.

Structured programming is a programming paradigm aimed at improving the **clarity**, quality of a computer program by making extensive use of the **structured control flow constru** and repetition (while and for), block structures, and subroutines. It is possible to do structured programming in any $n = 1$ programming language, though it is preferable to use something like a procedural progr

Procedural programming is a programming paradigm, derived from *imperative produced* concept of the *procedure call*. Procedures (a type of routine or subroutine) simply containery steps to be carried out. Any given procedure might be called at any point during a prog other procedures or itself. The first major procedural programming languages including Fortran, ALGOL, COBOL, PL/I and BASIC.^[2] Pascal and C were published circa 19

The example is on the next slide and is from Rober **book: Agile Software Development - Principles, Patterr**

```
/**
* This class Generates prime numbers up to a user specified
* maximum. The algorithm used is the Sieve of Eratosthenes.
* <p>
* Eratosthenes of Cyrene, b. c. 276 BC, Cyrene, Libya --
* d. c. 194, Alexandria. The first man to calculate the
* circumference of the Earth. Also known for working on
* calendars with leap years and ran the library at Alexandria.
* <p>
* The algorithm is quite simple. Given an array of integers
* starting at 2. Cross out all multiples of 2. Find the next
* uncrossed integer, and cross out all of its multiples.
 * Repeat until you have passed the square root of the maximum
 * value.
 * @author Robert C. Martin
 * @version 9 Dec 1999 rcm
*/
public class GeneratePrimes
 /**
   * @param maxValue is the generation limit.
  */
 public static int[] generatePrimes(int maxValue)
  {
    …
  }
While this program makes extensive
use of the control flow constructs of
                                        Principles, Patterns, and Practices
structured programming, it makes
very little use of subroutines.
```
*

{

}

public static int[] **generatePrimes**(**int** maxValue) { **if** (maxValue $>= 2$) { $//$ the only valid case // declarations **int** $s = maxValue + 1$; // size of array **boolean**[] f = **new boolean**[s]; **int** i; // initialize array to true. **for** $(i = 0; i < s; i++)$ f[i] = **true**; // get rid of known non-primes f[0] = f[1] = **false**; // sieve **int** j; **for** $(i = 2; i < Math.sqrt(s) + 1; i++)$ { **if** (f[i]) { // if i is uncrossed, cross its multiples. **for** $(i = 2 * i; j < s; j == i)$ f[j] = **false**; // multiple is not prime } } // how many primes are there? int $count = 0$; **for** $(i = 0; i < s; i++)$ { **if** (f[i]) count++; // bump count. } **int**[] primes = **new int**[count]; // move the primes into the result **for** $(i = 0, j = 0; i < s; i++)$ { **if** (f[i]) // if prime $primes[j++] = i;$ } **return** primes; // return the primes } **else** // maxValue < 2 **return new int**[0]; // return null array if bad input.

}

Robert C. Martin

This program generates **prime numbers**.

It is **one big function** with many **single letter variables** and **comments** to **help** us read it.

Notice that the generatePrimes function is divided into sections such as *declarations***,** *initializations***, and** *sieve*.

This is an obvious symptom of doing more than one thing.

Functions that do one thing cannot be reasonably divided into sections.

Instead of showing you the test code associated with that program, here is a simple test demonstrating that the program correctly computes the **primes** in the range 2..30.

```
public class Main {
 public static void main(String[] args) {
   int[] actualPrimes = GeneratePrimes.generatePrimes(30);
   int[] expectedPrimes = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29};
   if (!Arrays.equals(actualPrimes, expectedPrimes))
     throw new AssertionError(
        "GeneratePrimes.generatePrimes(30) returned " + Arrays.toString(actualPrimes));
  }
}
```


Robert Martin @unclebobmartin I wrote the module … for the first *XP Immersion*.

It was intended to be an example of bad coding and commenting style.

Kent Beck then refactored this code into a much more pleasant form in front of several dozen enthusiastic students.

Later I adapted the example for my book *Agile Software Development, Principles, Patterns, and Practices* and the first of my *Craftsman* articles published in *Software Development* magazine.

What I find fascinating about this module is that there was a time when many of us would have **considered it** "**well documented**."

Now we see it as a small mess.

See how many different comment problems you can find.

Jeff Langr @jlangr

Comments indicate that code is not communicating clearly.

As others have said, "Comments are lies."

You can't trust comments; the only thing you can truly depend on is working code.

Comments are often used as a deodorant for the rotten whiffs of bad code.

Strive to eliminate comments in your code.

becomes superfluous.

Martin Fowler @martinfowler

Robert Martin @unclebobmartin **The proper use of comments is to compensate for our failure to express ourself in code.**

Note that I used the word *failure*. I meant it. Comments are always failures. We must have them **because we cannot always figure out how to express ourselves without them, but their use is not a cause for celebration.**

When you feel the need to write a comment, first try to refactor the code so that any comment

So when you find yourself in a position where you need to write a comment, think it through and **see whether there isn't some way to turn the tables and express yourself in code.**

Every time you express yourself in code, you should pat yourself on the back. Every time you write a **comment, you should grimace and feel the failure of your ability of expression.**

Clean Code

Method Comment Pattern

- **How do you comment methods?**
- **Communicate important information that is not obvious from the code in a comment at the beginning of the method**
- **I expect you to be skeptical about this pattern**
- **Experiment**:
	- **Go through your methods and delete only those comments that duplicate exactly what the code says**
	- **If you can't delete a comment, see if you can refactor the code using these patterns (…) to communicate the same thing**
- **I will be willing to bet that when you are done you will have almost no comments left**

Of course the above is just a summary of the pattern.

Functions Should Do One Thing

}

}

It is often tempting to create functions that have multiple sections that perform a series of operations. **Functions of this kind do more than** *one thing***, and should be converted into many smaller functions, each of which does** *one thing***.** For example:

```
public void pay() { 
    for (Employee e : employees) { 
        if (e.isPayday()) { 
            Money pay = e.calculatePay(); 
            e.deliverPay(pay);
         } 
     }
```
This bit of code does three things. **It loops over all the employees, checks to see whether each employee ought to be paid, and then pays the employee. This code would be better written as:**

```
public void pay() { 
    for (Employee e : employees) 
        payIfNecessary(e); 
}
```

```
private void payIfNecessary(Employee e) { 
    if (e.isPayday()) 
        calculateAndDeliverPay(e);
}
```

```
private void calculateAndDeliverPay(Employee e) { 
    Money pay = e.calculatePay(); 
    e.deliverPay(pay);
```



```
Robert Martin
@unclebobmartin
```


Understand the Algorithm

Lots of very funny code is written because people don't take the time to understand the algorithm.

They get something to work by plugging in enough if statements and flags, without really stopping to consider what is really going on.

Programming is often an exploration.

You *think* **you know the right algorithm for something, but then you wind up fiddling with it, prodding and poking at it, until you get it to "work."**

How do you know it "works"? Because it passes the test cases you can think of.

There is nothing wrong with this approach.

Indeed, often it is the only way to get a function to do what you think it should.

However, it is not sufficient to leave the quotation marks around the word "work."

Before you consider yourself to be done with a function, make sure you *understand* **how it works.**

It is not good enough that it passes all the tests. You must *know***¹⁰ that the solution is correct.**

Often the best way to gain this knowledge and understanding is to refactor the function into something that is so clean and expressive that it is *obvious* **how it works.**

10. There is a difference between **knowing how the code works** and knowing whether the algorithm will do the job required of it. Being unsure that an algorithm is appropriate is often a fact of life. **Being unsure what your code does is just laziness**.

Robert Martin @unclebobmartin

The **prime generator** is hard to **understand** and **maintain** because it consists of a **single method** that **does more than one thing**.

The method is divided into **sections** (each signposted by a **comment**), with each **section** doing one of the **multiple things**.

On the next slide, **Uncle Bob** asserts that if a program is **difficult** to **understand** and/or **change** then it is **broken** and **needs fixing**.

Every software module has three functions.

First is the function it

for the module's existence.

It takes attention.

It takes discipline.

It takes a passion for creating beauty.

The second function of a module is to afford change.

Almost all modules will change in the course of their lives, and it is the responsibility of the developers to make sure that such changes are as simple as possible to make.

A module that is difficult to change is broken and needs fixing, even though it works.

The third function of a module is to communicate to its readers.

Developers who are not familiar with the module should be able to read and understand it without undue mental gymnastics.

A module that does not communicate is broken and needs to be fixed.

What does it take to make a module easy to read and easy to change?

Much of this book is dedicated to principles and patterns whose primary goal is to help you create modules that are flexible and adaptable.

But it takes something more than just principles and patterns to make a module that is easy to read and change.

What we are going to do next is follow **Uncle Bob** as he **refactors** the **prime generator** so that it becomes **easier** to **understand** and to **change**.

```
public static int[] generatePrimes(int maxValue) 
                                                                        {
                                                                          if (maxValue >= 2) { // the only valid case
                                                                            // declarations
                                                                            int s = maxValue + 1; // size of array
                                                                            boolean[] f = new boolean[s];
                                                                            int i;
                                                                            // initialize array to true.
                                                                            for (i = 0; i < s; i++)f[i] = true;
                                                                            // get rid of known non-primes
                                                                            f[0] = f[1] = false;
                                                                            // sieve
                                                                            int j;
                                                                            for (i = 2; i < Math.sqrt(s) + 1; i++) {
                                                                              if (f[i]) { // if i is uncrossed, cross its multiples.
                                                                                for (j = 2 * i; j < s; j += i)f[j] = false; // multiple is not prime
                                                                               }
                                                                            }
                                                                            // how many primes are there?
                                                                            int count = \theta;
                                                                            for (i = 0; i < s; i++) {
                                                                              if (f[i])
                                                                                count++; // bump count.
                                                                             }
                                                                            int[] primes = new int[count];
                                                                            // move the primes into the result
                                                                            for (i = 0, j = 0; i < s; i++) {
                                                                              if (f[i]) // if prime
                                                                                primes[j++) = i;}
                                                                            return primes; // return the primes
                                                                          } else // maxValue < 2
                                                                            return new int[0]; // return null array if bad input.
                                                                        }
 * This class Generates prime numbers up to a user specified
 * maximum. The algorithm used is the Sieve of Eratosthenes.
 * Eratosthenes of Cyrene, b. c. 276 BC, Cyrene, Libya --
 * d. c. 194, Alexandria. The first man to calculate the
 * circumference of the Earth. Also known for working on
 * calendars with leap years and ran the library at Alexandria.
 * The algorithm is quite simple. Given an array of integers
 * starting at 2. Cross out all multiples of 2. Find the next
 * uncrossed integer, and cross out all of its multiples.
 * Repeat until you have passed the square root of the maximum
 * @author Robert C. Martin
 * @version 9 Dec 1999 rcm
public class GeneratePrimes
   * @param maxValue is the generation limit.
  public static int[] generatePrimes(int maxValue)
             It seems pretty clear that the main function wants to be
             three separate functions.
             The first initializes all the variables and sets up the sieve.
             The second executes the sieve, and the third loads the
             sieved results into an integer array.
```
Robert Martin @unclebobmartin

/**

 $*$ <p>

 $*$ $\langle p \rangle$

* value.

*

*/

*/

{ … } }

{ /**

```
public static int[] generatePrimes(int maxValue) 
{
 if (maxValue >= 2) { // the only valid case
    // declarations
   int s = maxValue + 1; // size of array
   boolean[] f = new boolean[s];
   int i;
   // initialize array to true.
   for (i = 0; i < s; i++)f[i] = true;
   // get rid of known non-primes
   f[0] = f[1] = false;
   // sieve
   int j;
   for (i = 2; i < Math.sqrt(s) + 1; i++) {
     if (f[i]) { // if i is uncrossed, cross its multiples.
       for (j = 2 * i; j < s; j == i)f[j] = false; // multiple is not prime
      }
    }
   // how many primes are there?
   int count = \theta;
   for (i = 0; i < s; i++) {
     if (f[i])
        count++; // bump count.
    }
   int[] primes = new int[count];
   // move the primes into the result
   for (i = \theta, j = \theta; i < s; i++) {
     if (f[i]) // if prime
        primes[j++) = i;}
   return primes; // return the primes
  } else // maxValue < 2
   return new int[0]; // return null array if bad input.
}
                                                                                                                             public static int[] generatePrimes(int maxValue) 
                                                                                                                             {
                                                                                                                               if (maxValue < 2) {
                                                                                                                                 return new int[0];
                                                                                                                               else {
                                                                                                                                 initializeSieve(maxValue);
                                                                                                                                 sieve();
                                                                                                                                 loadPrimes();
                                                                                                                                 return primes;
                                                                                                                               }
                                                                                                                             }
                                                                    private static void initializeSieve(int maxValue)
                                                                    {
                                                                      // declarations
                                                                      s = maxValue + 1; // size of array
                                                                      f = new boolean[s];
                                                                      int i;
                                                                      // initialize array to true.
                                                                      for (i = 0; i < s; i++)f[i] = true;
                                                                      // get rid of known non-primes
                                                                      f[0] = f[1] = false;
                                                                    }
                                                                    private static void sieve()
                                                                    {
                                                                      int i;
                                                                      int j;
                                                                      for (i = 2; i < Math.sqrt(s) + 1; i++)if (f[i]) // if i is uncrossed, cross its multiples.
                                                                        {
                                                                          for (i = 2 * i; j < s; j == i)f[j] = false; // multiple is not prime
                                                                        }
                                                                      }
                                                                    }
                                                                                                                                         private static void loadPrimes() 
                                                                                                                                          {
                                                                                                                                           int i;
                                                                                                                                           int j;
                                                                                                                                           // how many primes are there?
                                                                                                                                           int count = 0;
                                                                                                                                           for (i = 0; i < s; i++)if (f[i])
                                                                                                                                               count++; // bump count.
                                                                                                                                            }
                                                                                                                                           primes = new int[count];
                                                                                                                                           // move the primes into the result
                                                                                                                                           for (i = 0, j = 0; i < s; i++)if (f[i]) // if prime
                                                                                                                                               primes[j++] = i;}
                                                                                                                                          }
                                                                    public class PrimeGenerator
                                                                    {
                                                                      private static int s;
                                                                      private static boolean[] f;
                                                                      private static int[] primes;
                                                                      public static int[] generatePrimes(int maxValue)
                                                                      {
                                                                        …
                                                                      }
                                                                    }
                                                                                                                              To expose this structure more clearly, I extracted those
                                                                                                                              functions into three separate methods.
                                                                                                                              I also removed a few unnecessary comments and
                                                                                                                              changed the name of the class to PrimeGenerator.
                                                                                                                              The tests all still ran.
                                                                                                           Robert Martin
                                                                                                       @unclebobmartin
                                                                                                   Extracting the three functions forced
                                                                                                   me to promote some of the variables
                                                                                                   of the function to static fields of the
                                                                                                   class. This makes it much clearer which
                                                                                                   variables are local and which have
                                                                                                   wider influence.
```


Next, I looked at **crossOutMultiples**. There were a number of statements in this function, and in others, of the form **if(f[i] == true)**. The intent was to check whether **i** was **uncrossed**, so I changed the name of **f** to **unCrossed**. But this led to ugly statements, such as **unCrossed[i] = false**. I found the **double negative** confusing. So I changed the name of the array to **isCrossed** and changed the sense of all the Booleans. The tests all still ran.

I got rid of the initialization that set **isCrossed[0]** and **isCrossed[1]** to true and simply made sure that no part of the function used the **isCrossed** array for indexes less than 2. I extracted the inner loop of the **crossOutMultiples** function and called it **crossOutMultiplesOf**. I also thought that **if (isCrossed[i] == false)** was confusing, so I created a function called **notCrossed** and changed the **if** statement to **if (notCrossed(i))**. The tests all still ran.

private static void crossOutMultiples() { **int** i; **int** j; **for** $(i = 2; i < 1)$ Math.sqrt(f.length) + 1; i++){ **if** ($f[i]$) // if i is uncrossed, cross its multiples. { **for** $(j = 2 * i; j < f.length; j += i)$ f[j] = **false**; // multiple is not prime } } }

private static void initializeArrayOfIntegers(**int** maxValue) { f = **new boolean**[maxValue + 1]; f[0] = f[1] = **false**; // neither primes nor multiples **for** ($int i = 2$; $i \lt f.length$; $i++)$ f[i] = **true**; }

Robert Martin @unclebobmartin

I spent a bit of time writing a comment that tried to explain why you have to iterate only up to the **square root** of the array size. This led me to extract the calculation into a function where I could put the **explanatory comment**. In writing the **comment**, I realized that the **square root** is the **maximum prime factor** of any of the integers in the array. So I chose that name for the variables and functions that dealt with it. The result of all these refactorings are [on the next page] ... The tests all still ran.

```
private static void crossOutMultiples()
{
 int i;
  int j;
 for (i = 2; i < Math.sqrt(f.length) + 1; i++)if (f[i]) // if i is uncrossed, cross its multiples.
    {
     for (j = 2 * i; j < f.length; j += i)f[j] = false; // multiple is not prime
    }
  }
}
```


The last function to refactor is **PutUncrossedIntegersIntoResult**. This method has **two parts**.

The **first** counts the number of uncrossed integers in the array and creates the result array of that size.

The **second** moves the uncrossed integers into the result array.

I extracted the first part into its own function and did some **miscellaneous cleanup**.

The tests all still ran.

@unclebobmartinV

Next, I made one final pass over the whole program, reading it from beginning to end, rather like one would read a geometric proof.

This is an important step.

So far, I've been refactoring fragments.

Robert Martin Now I want to see whether the whole program hangs together as a *readable* whole.

First, I realize that I don't like the name **InitializeArrayOfIntegers**.

What's being initialized is not, in fact, an array of integers but an array of Booleans.

But **InitializeArrayOfBooleans** is not an improvement.

What we are really doing in this method is **uncrossing** all the relevant integers so that we can then **cross out** the **multiples**.

So I change the name to **uncrossIntegersUpTo**.

I also realize that I don't like the name **isCrossed** for the array of Booleans.

So I change it to **crossedOut**.

The tests all still run.

```
private static void 
initializeArrayOfIntegers(int maxValue)
{
  isCrossed = new boolean[maxValue + 1];
  for (int i = 2; i < isCrossed.length; i++)isCrossed[i] = false;
}
```


One might think that I'm being **frivolous** with these **name changes**, but with a refactoring browser, you can afford to do these kinds of **tweaks**; they cost virtually nothing.

Even without a refactoring browser, a simple search and replace is pretty cheap.

And the tests strongly mitigate any chance that we might unknowingly break something.

}

I don't know what I was smoking when I wrote all that **maxPrimeFactor** stuff.

Yikes! The **square root** of the size of the array is not necessarily **prime**.

That method did *not* calculate the **maximum prime factor**.

The explanatory comment was simply *wrong*.

So I rewrote the comment to better explain the rationale behind the square root and rename all the variables appropriately.

{

}

The tests all still run.

```
private static void crossOutMultiples()
{
 int maxPrimeFactor = calcMaxPrimeFactor();
 for (int i = 2; i \leq maxPrimeFactor; i++)if (notCrossed(i))
      crossOutMultiplesOf(i);
}
```
private static void crossOutMultiples() { **int** limit = **determineIterationLimit**(); **for** (int $i = 2$; $i \leq \frac{limit}{i}$; $i++)$ **if** (**notCrossed**(i)) **crossOutMultiplesOf**(i); }

// We cross out all multiples of p, where p is prime. // Thus, all crossed out multiples have p and q for // factors. If $p >$ sqrt of the size of the array, then // q will never be greater than 1. Thus p is the // largest prime factor in the array and is also // the iteration limit. **private static int calcMaxPrimeFactor**() { **double** maxPrimeFactor = Math.sqrt(crossedOut.**length**) + 1; **return** (**int**) maxPrimeFactor;

double iterationLimit = Math.sqrt(crossedOut.**length**) + 1; **return** (**int**) iterationLimit;

What the devil is that +1 doing in there?

Robert Martin @unclebobmartin

It must have been paranoia.

I was afraid that a **fractional square root** would convert to an integer that was too small to serve as the iteration limit.

But that's silly.

The true iteration limit is the largest **prime** less than or equal to the **square root** of the size of the array.

}

I'll get rid of the +1.

The tests all run, but that last change makes me pretty nervous.

I understand the rationale behind the **square root**, but I've got a nagging feeling that there may be some **corner cases** that aren't being **covered**.

So I'll write another test that checks that there are no **multiples** in any of the **prime** lists between 2 and 500. …

The new test passes, and my fears are allayed.

/** * This class Generates prime numbers up to a user specified * maximum. The algorithm used is the Sieve of Eratosthenes. * Given an array of integers starting at 2: * Find the first uncrossed integer, and cross out all its * multiples. Repeat until there are no more multiples * in the array. */ **public class PrimeGenerator** { **private static boolean**[] crossedOut; **private static int**[] result; **public static int**[] **generatePrimes**(**int** maxValue) { **if** (maxValue $\langle 2 \rangle$ { **return new int**[0]; **else** { **uncrossIntegersUpTo**(maxValue); **crossOutMultiples**(); **putUncrossedIntegersIntoResult**(); **return** result; } } } **private static void uncrossIntegersUpTo**(**int** maxValue) { crossedOut = **new boolean**[maxValue + 1]; **for** ($int i = 2$; $i \lt c$ rossedOut. length; $i++)$ crossedOut[i] = **false**; } **private static void crossOutMultiples**() { **int** limit = **determineIterationLimit**(); **for** ($int i = 2$; $i \le limit; i++)$ **if** (**notCrossed**(i)) **crossOutMultiplesOf**(i); } // Every multiple in the array has a prime factor that // is less than or equal to the root of the array size, // so we don't have to cross off multiples of numbers // larger than that root. **private static int determineIterationLimit**() { **double** iterationLimit = Math.sqrt(crossedOut.**length**); **return** (**int**) iterationLimit; } **private static void putUncrossedIntegersIntoResult** () { result = **new int**[**numberOfUncrossedIntegers**()]; **for** (int $i = 2$, $j = 0$; $i \lt c$ rossedOut.length; $i++)$ **if** (**notCrossed**(i)) $result[j++] = i;$ } **private static int numberOfUncrossedIntegers**() { int count = θ ; **for** ($int i = 2$; $i \lt c$ rossedOut.length; $i++)$ **if** (**notCrossed**(i)) count++; **return** count; } **private static boolean notCrossed**(**int** i) { **return** crossedOut[i] == **false**; } **private static void crossOutMultiplesOf**(**int** i) { **for** (int multiple = $2 * i$; multiple < crossedOut.**length**; $multiple += i)$ crossedOut[multiple] = **true**; } **Robert Martin @unclebobmartin Note that the use of comments is significantly restrained**. **There are just two comments** i**n the whole module. Both comments are explanatory in nature. The end result of this program reads much better than it did at the start.** It also **works a bit better**. I'm pretty pleased with the outcome. **The program is much easier to understand and is therefore much easier to change**. **Also, the structure of the program has isolated its parts from one another. This also makes the program much easier to change**.

BEFORE AFTER AND A SERIES AND A

The original **sieve** program was developed using **imperative programming** and, except for the fact that it consisted of a single method, **structured programming**. It was then **refactored**, using **functional decomposition**, into a **more understandable** and **maintainable** program which, consisting of several methods, could now more legitimately be considered an example of **structured**/**procedural programming**.

What we are going to do next is look at a **sieve** program developed using the following:

- 1. The **immutable FP** data structure of a **sequence**, implemented using a **list**
- 2. The basic **sequence operations** to
	- **Construct** a **sequence**
	- Get the **first** element of a **sequence**
	- Get the **rest** of a **sequence**
- 3. A **filter** function that given a **sequence** and a **predicate** (a function that given a value, returns true if the value satisfies the predicate and false otherwise), returns a new **sequence** by selecting only those elements of the original **sequence** that satisfy the **predicate**.

What we'll find is that **using these simple building blocks it is possible to write a sieve program which is so simple that it is much easier to understand and maintain than the procedural one we have just seen**.

By the way, don't assume that **Uncle Bob** isn't interested in alternative ways of implementing the **Sieve of Eratosthenes**, as we shall see in part 2.

This simpler **sieve** program is described in **Structure and Interpretation of Computer Programs** (**SICP**) using **Scheme**.

The following two slides are a lightning-fast, extremely minimal refresher on the **building blocks** used to develop the program.

If you are completely new to **immutable data structures**, **sequences**, and **filtering**, and could do with an introduction to them, then why not catch up using the slide deck below?

Structure and Interpretation of Computer **Programs Second Edition Harold Abelson and Gerald Jay Sussman** with Julie Sussman

SICP

Constructing a sequence

E.

What I said earlier wasn't the full truth: the **sieve** program in **SICP** uses not just plain **sequences**, implemented using **lists**, but ones implemented using **streams**, i.e. **lazy** and possibly **infinite** sequences.

An introduction to **streams** is outside the scope of this deck so let's learn (or review) just enough about **streams** to be able to understand the **SICP sieve** program, so that we can then convert the program to use **lists** rather than **streams**.

Streams are a clever idea that allows one to use sequence manipulations without incurring the costs of manipulating sequences **as lists**.

With streams we can achieve the best of both worlds: We can formulate programs elegantly as sequence manipulations, while **attaining the efficiency of incremental computation**.

The basic idea is to arrange to construct a stream only partially, and to pass the partial construction to the program that **consumes the stream.**

If the consumer attempts to access a part of the stream that has not yet been constructed, the stream will automatically construct just enough more of itself to produce the required part, thus preserving the illusion that the entire stream exists.

In other words, although we will write programs as if we were processing complete sequences, we design our stream **implementation to automatically and transparently interleave the construction of the stream with its use.**

On the surface, streams are just lists with different names for the procedures that manipulate them.

There is a constructor, cons-stream, and two selectors, stream-car and stream-cdr, which satisfy the constraints

```
(stream-car (cons-stream x y)) = x
```

```
(stream-cdr (cons-stream x y)) = y
```
There is a distinguishable object, the-empty-stream, which cannot be the result of any cons-stream operation, and which can be identified with the predicate stream-null?. Thus we can make and use streams, in just the same way as we can make and **use lists, to represent aggregate data arranged in a sequence.**

Interpretation of Computer Programs

To compute the **prime numbers**, we take the **infinite stream** (**define primes** (sieve (integers-starting-from 2))) of integers from 2 onwards and pass them through a **sieve**.

Sieving a **stream** of integers so that we only keep those that are **prime** numbers is done by constructing a **new stream** as follows:

- 1. The **head** of the **sieved stream** is the **head** of the **incoming stream**. Let's refer to this as the **next prime number**. See next slide for why this is a **prime number**.
- 2. The **tail** of the **sieved stream** is created by **recursively sieving** a **new stream** which is the result of taking the **tail** of the **incoming stream** and then **filtering out** any integers which are **multiples** of the **next prime number** and which are therefore **not prime**.

The way the **sieve** function works out which integers are **multiples** of the **next prime number** (so that it can filter them out), is by using the **divisible?** function to check that they are not **divisible** by the **prime number**.

(**define** (**divisible?** x y) (= (**remainder** x y) 0))

As for the infinite integers from 2 onwards, they are defined **recursively**.

(**define** (**integers-starting-from** n) (**cons-stream** n (**integers-starting-from** (+ n 1))))

We start with the integers beginning with 2, which is the **first prime**.

To get the rest of the **primes**, we start by **filtering** the **multiples** of 2 from the **rest** of the integers.

This leaves a **stream** beginning with 3, which is the **next prime**.

Now we filter the **multiples** of 3 from the rest of this **stream**.

This leaves a **stream** beginning with 5, which is the **next prime**, and so on. In other words, we construct the **primes** by a **sieving process**, described as follows: To **sieve** a **stream S**, form a **stream** whose first element is the first element of **S** and the rest of which is obtained by **filtering** all **multiples** of the first element of **S** out of the rest of **S** and **sieving** the result.

This process is readily described in terms of **stream operations**:

```
(define (sieve stream)
 (cons-stream
   (stream-car stream)
   (sieve (stream-filter
             (lambda (x)(not (divisible? x (stream-car stream))))
             (stream-cdr stream)))))
```

```
(define primes (sieve (integers-starting-from 2)))
```
Now to find a particular **prime** we need only ask for it:

```
scheme> (stream-ref primes 50)
233
```

```
(define (stream-ref s n)
  (if (= n 0)
      (stream-car s)
      (stream-ref (stream-cdr s) (- n 1))))
```


It is interesting to contemplate the **signal-processing system** set up by **sieve**, shown in the "**Henderson diagram**" in figure 3.32.

The **input stream** feeds into an "**unconser**" that separates the **first** element of the **stream** from the **rest** of the **stream**.

The first element is used to construct a **divisibility filter**, through which the **rest** is passed, and the **output** of the filter is fed to **another sieve box**.

Then the original **first element** is **consed** onto the **output** of the **internal sieve** to form the **output stream**.

Thus, not only is the **stream infinite**, but the **signal processor** is also **infinite**, because the **sieve** contains a **sieve** within it.

Programs must be written for people to read, and only incidentally for machines to execute.

The key to understanding complicated things is knowing what not to look at.

Harold Abelson **Gerald Jay Sussman**

Moulube MIT 6.001 Structure and Interpretation, 1986

Because the **stream** of **primes** is **infinite**, some auxiliary functions are needed to display a subset of them.

```
(define (stream-take stream n)
 (if (= n 0)
   nil
   (cons-stream
      (stream-car stream)
     (stream-take (stream-cdr stream) (- n 1)))))
(define (display-stream s)
 (stream-for-each display-line s))
(define (display-line x)
 (newline)
 (display x))
```



```
/**
* This class Generates prime numbers up to a user specified
* maximum. The algorithm used is the Sieve of Eratosthenes.
* Given an array of integers starting at 2:
* Find the first uncrossed integer, and cross out all its
* multiples. Repeat until there are no more multiples
* in the array.
 */
public class PrimeGenerator
{
 private static boolean[] crossedOut;
 private static int[] result;
 public static int[] generatePrimes(int maxValue) 
  {
    …
  }
}
```


On the next slide we take the **Scheme sieve** program and modify it as follows:

- get the program to operate on **finite lists** rather than **infinite streams**.
- replace the **primes** function with with a **generatePrimes** function.

(**define** (**integers-starting-from** n) (**cons-stream** n (**integers-starting-from** (+ n 1))))

(**define** (**enumerate-interval** low high) (**if** (> low high) **nil** (**cons** low (**enumerate-interval** (+ low 1) high))))

(**define** (**sieve** candidates) (**if** (**null?** candidates) **nil** (**cons** (**car** candidates) (**sieve** (**filter** (**define** (**generate-primes** maxValue) (if (< maxValue 2) **nil** (**sieve** (**enumerate-interval** 2 maxValue))) (**define** (**enumerate-interval** low high) (**if** (> low high) **nil** (**cons** low (**enumerate-interval** (+ low 1) high)))) Here is the resulting program again. Let's take it for a spin.

> (**define** (**divisible?** x y) (= (**remainder** x y) 0))

scheme> (**sieve** '(2 3 4 5 6 7 8 9 10)) (2 3 5 7)

scheme> (**enumerate-interval** 2 10) (2 3 4 5 6 7 8 9 10)

(**lambda** (x)(**not** (**divisible?** x (**car** candidates))))

(**cdr** candidates))))))

scheme> (**sieve** (**enumerate-interval** 2 10)) (2 3 5 7)

scheme> (**generate-primes** 10) (2 3 5 7)

Here is the **Haskell**, program again, after inlining the **enumerateInterval** function. Let's take it for a spin.

```
generatePrimes :: Int -> [Int]
generatePrimes maxValue =
  if maxValue < 2
 then []
  else sieve [2..maxValue]
```

```
sieve :: [Int] -> [Int]
sieve [] = []
sieve (nextPrime:candidates) = 
  nextPrime : sieve noFactors
  where noFactors = filter (not . (`divisibleBy` nextPrime)) 
                           candidates
```
divisibleBy :: **Int** -> **Int** -> **Bool divisibleBy** $x \ y = \text{mod} \ x \ y = 0$

Haskell is **lazy** e.g. its **lists** can be **infinite**, so just like we did in **Scheme** with **streams**, we could define the **infinite** list of **primes** as the **sieve** of the **infinite list** of integers starting from 2, and then operate on the **primes**.

\sum

(**define primes** (**sieve** (**integers-starting-from** 2))) **primes** = **sieve** [2..]

haskell> **take** 10 **primes** [2,3,5,7,11,13,17,19,23,29]

scheme> (**stream-ref primes** 50) 233

\sum

generatePrimes :: **Int** -> [**Int**] **generatePrimes** maxValue = **if** maxValue < 2 **then** [] **else sieve** [2..maxValue]

def generatePrimes(maxValue: **Int**): **List**[**Int**] = **if** maxValue < 2 **then Nil else sieve**(**List**.range(2,maxValue + 1))

```
sieve :: [Int] -> [Int]
sieve [] = []
sieve (nextPrime:candidates) = 
  nextPrime : sieve noFactors
  where noFactors = filter (not . (`divisibleBy` nextPrime)) 
                           candidates
```


divisibleBy :: **Int** -> **Int** -> **Bool divisibleBy** $x \ y = \text{mod} \ x \ y = 0$

extension (m: **Int**) **def divisibleBy**(n: Int): **Boolean** = $m \times n = 0$


```
def generatePrimes(maxValue: Int): List[Int] =
  if maxValue < 2
  then Nil
  else sieve(List.range(2,maxValue + 1))
```

```
def sieve(candidates: List[Int]): List[Int] = candidates match
  case Nil => Nil
  case nextPrime :: rest =>
    val nonMultiples = rest filterNot (_ divisibleBy nextPrime)
    nextPrime :: sieve(nonMultiples)
```
extension (m: **Int**) **def divisibleBy**(n: **Int**): **Boolean** = m % n == 0

```
scala> generatePrimes(30)
val res0: List[Int] = List(2, 3, 5, 7, 11, 13, 17, 19, 23, 29)
```


In the next two slides, let's compare the **Scala** program with the **Java** program, both before and after refactoring the latter.

```
public static int[] generatePrimes(int maxValue) 
{
 if (maxValue >= 2) { // the only valid case
   // declarations
   int s = maxValue + 1; // size of array
   boolean[] f = new boolean[s];
   int i;
   // initialize array to true.
   for (i = 0; i < s; i++)f[i] = true;
   // get rid of known non-primes
   f[0] = f[1] = false;
   // sieve
   int j;
   for (i = 2; i < Math.sqrt(s) + 1; i++) {
     if (f[i]) { // if i is uncrossed, cross its multiples.
       for (j = 2 * i; j < s; j == i)f[j] = false; // multiple is not prime
     }
    }
   // how many primes are there?
   int count = \theta;
   for (i = 0; i < s; i++) {
     if (f[i])
       count++; // bump count.
    }
   int[] primes = new int[count];
   // move the primes into the result
   for (i = 0, j = 0; i < s; i++) {
     if (f[i]) // if prime
       primes[j++] = i;}
   return primes; // return the primes
 } else // maxValue < 2
   return new int[0]; // return null array if bad input.
}
```
FP immutable sequence and filtering

If I have to choose which of the two programs I'd rather have to **understand** and **maintain**, then I am compelled to pick the one on the right, due to its **succinctness**.

def generatePrimes(maxValue: **Int**): **List**[**Int**] = **if** maxValue < 2 **then Nil else sieve**(**List**.range(2, maxValue + 1))

def sieve(candidates: **List**[**Int**]): **List**[**Int**] = candidates **match case Nil** => **Nil case** nextPrime :: rest => **val** nonMultiples = rest **filterNot** (_ **divisibleBy** nextPrime) nextPrime :: **sieve**(nonMultiples)

extension (m: **Int**) **def divisibleBy**(n: **Int**): **Boolean** = $m % n = 0$

imperative and structured programming

