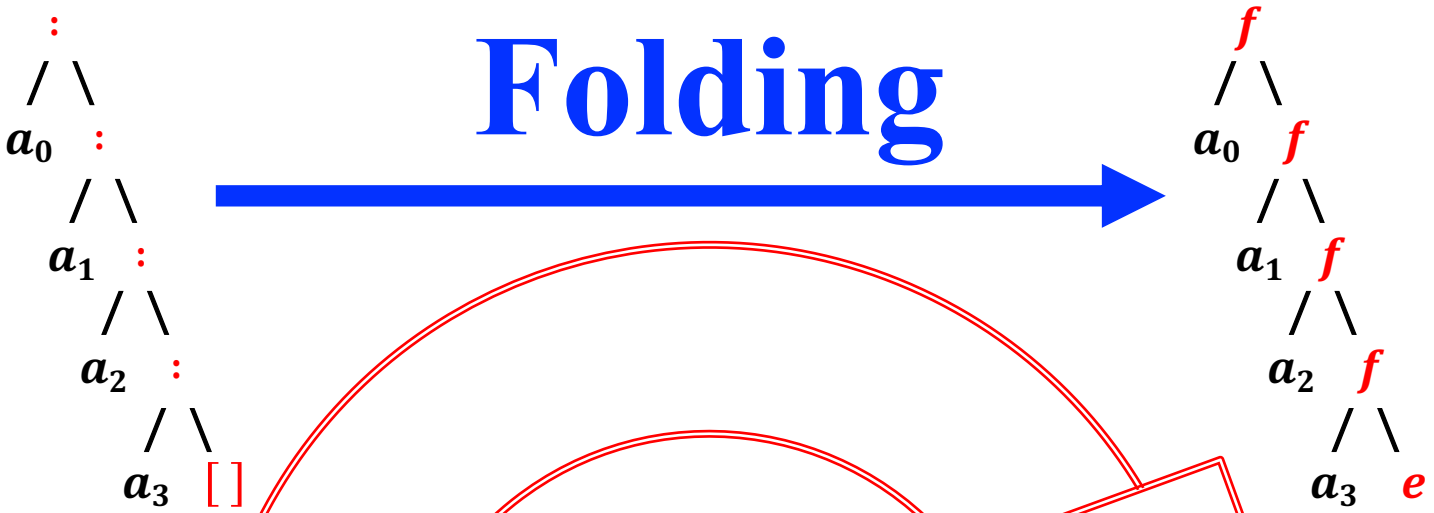



# Folding

CHEAT-SHEET

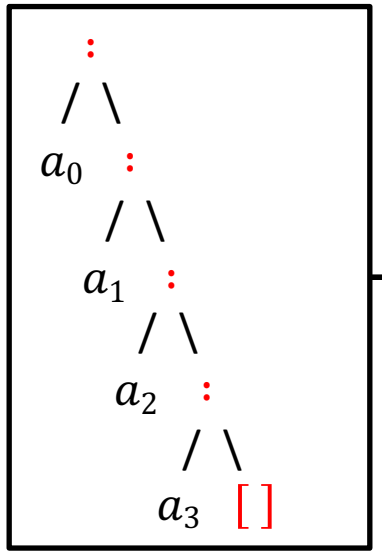
#2

slides by  @philip_schwarz FP Illuminated https://fpilluminated.com/

**Programmatic** definition of **right fold** and **left fold**

```
foldr                :: (α → β → β) → β → [α] → β
foldr f b []         = b
foldr f b (x:xs)     = f x (foldr f b xs)
```

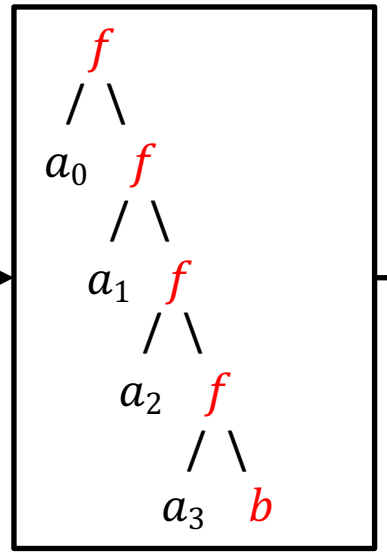
$as = [a_0, a_1, a_2, a_3]$




$a_0 : (a_1 : (a_2 : (a_3 : [\,])))$

*replace*:
: *with* $f$
[ ] *with* $b$

$foldr\ f\ b\ as$



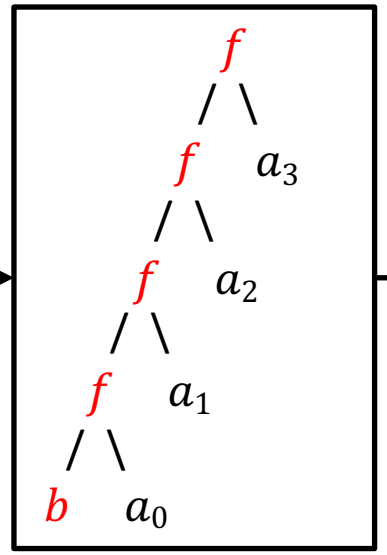

$f\ a_0\ (f\ a_1\ (f\ a_2\ (f\ a_3\ b)))$

$$
\begin{aligned}
&foldr\ f\ b\ [a_0, a_1, a_2, a_3] \\
&f\ a_0\ (foldr\ f\ b\ [a_1, a_2, a_3]) \\
&f\ a_0\ (f\ a_1\ (foldr\ f\ b\ [a_2, a_3])) \\
&f\ a_0\ (f\ a_1\ (f\ a_2\ (foldr\ f\ b\ [a_3]))) \\
&f\ a_0\ (f\ a_1\ (f\ a_2\ (f\ a_3\ (foldr\ f\ b\ [\,])))) \\
&f\ a_0\ (f\ a_1\ (f\ a_2\ (f\ a_3\ b)))
\end{aligned}
$$

*foldr* associates $f$ from the right

$foldl\ f\ b\ as$

```
var acc = b
foreach(a in as)
  acc = f(acc, a)
return acc
```




$f\ (f\ (f\ (f\ b\ a_0)\ a_1)\ a_2)\ a_3$

$$
\begin{aligned}
&foldl\ f\ b\ [a_0, a_1, a_2, a_3] \\
&foldl\ f\ (f\ b\ a_0)\ [a_1, a_2, a_3] \\
&foldl\ f\ (f\ (f\ b\ a_0)\ a_1)\ [a_2, a_3] \\
&foldl\ f\ (f\ (f\ (f\ b\ a_0)\ a_1)\ a_2)\ [a_3] \\
&foldl\ f\ (f\ (f\ (f\ (f\ b\ a_0)\ a_1)\ a_2)\ a_3)\ [\,] \\
&f\ (f\ (f\ (f\ b\ a_0)\ a_1)\ a_2)\ a_3
\end{aligned}
$$

*foldl* associates $f$ from the left

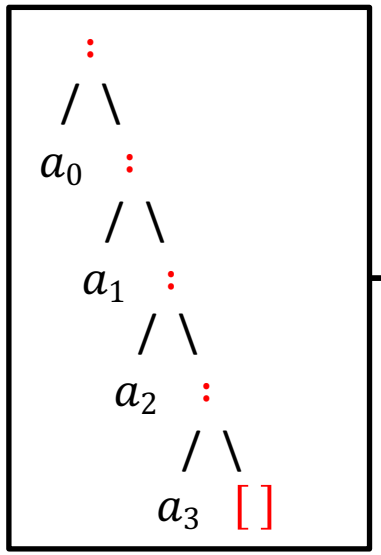
```
foldl                :: (β → α → β) → β → [α] → β
foldl f b []         = b
foldl f b (x:xs)     = foldl f (f b x) xs
```

**Mathematical** definition of **right fold** and **left fold**

$foldr([\,]) = b; \quad foldr([x] +\!\!+ xs) = f(x, foldr(xs))$

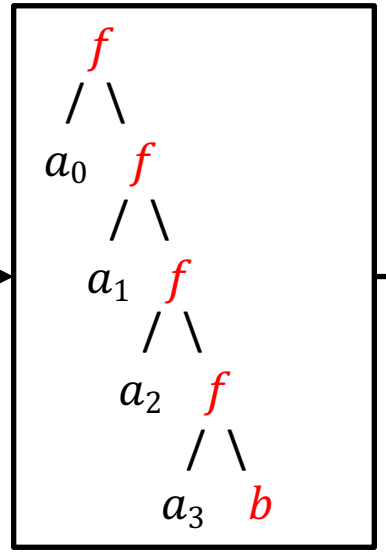
$x$ = the first element
$xs$ = all but the first element

$as = [a_0, a_1, a_2, a_3]$




$a_0 : (a_1 : (a_2 : (a_3 : [\,])))$

$foldr\ as$

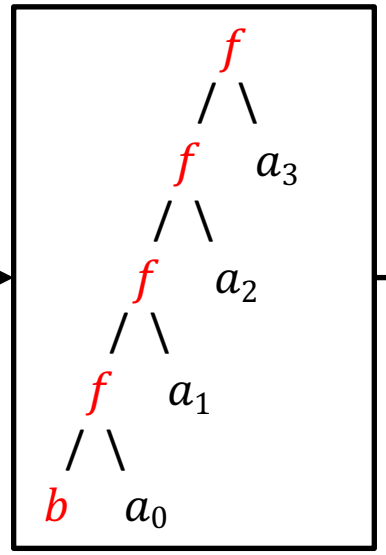



$f(a_0, f(a_1, f(a_2, f(a_3, b))))$

$foldr([a_0, a_1, a_2, a_3])$
$f(a_0, foldr([a_1, a_2, a_3]))$
$f(a_0, f(a_1, foldr([a_2, a_3])))$
$f(a_0, f(a_1, f(a_2, foldr([a_3]))))$
$f(a_0, f(a_1, f(a_2, f(a_3, foldr([])))))$
$f(a_0, f(a_1, f(a_2, f(a_3, b))))$

$foldr$ associates $f$ from the right

$foldl\ as$




$f(f(f(f(b, a_0), a_1), a_2), a_3)$

$foldl([a_0, a_1, a_2, a_3]),$
$f(foldl([a_0, a_1, a_2]), a_3)$
$f(f(foldl([a_0, a_1]), a_2), a_3)$
$f(f(f(foldl([a_0]), a_1), a_2), a_3)$
$f(f(f(f(foldl([\,]), a_0), a_1), a_2), a_3)$
$f(f(f(f(b, a_0), a_1), a_2), a_3)$

$foldl$ associates $f$ from the left

$x$ = the last element
$xs$ = all but the last element

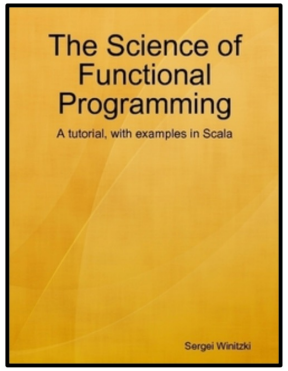
$foldl([\,]) = b; \quad foldl(xs +\!\!+ [x]) = f(foldl(xs), x)$

based on

# Left and Right Folds
# Comparison of a mathematical definition and a programmatic one

Polyglot FP for Fun and Profit - Haskell and Scala

Based on definitions from




Sergei Winitzki
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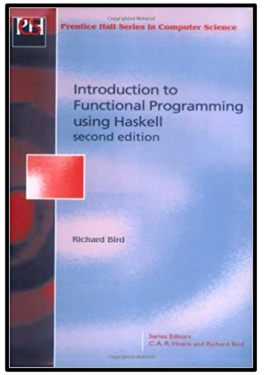


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